Parallelising Symbolic State-Space Generators:
Frustration & Hope

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In collaboration with

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Radu Siminiceanu, National Institute of Aerospace, Virginia

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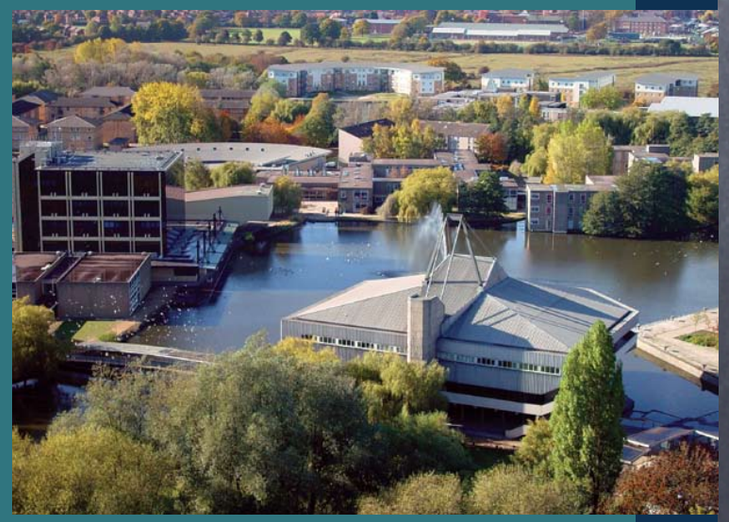
ETAPS 2009
22-29 MARCH
YORK ENGLAND

European Joint Conferences on Theory and Practice of Software

CC:	International Conference on Compiler Construction
ESOP:	European Symposium on Programming
FASE:	Fundamental Approaches to Software Engineering
FOSSACS:	Foundations of Software Science and Computation Structures
TACAS:	Tools and Algorithms for the Construction and Analysis of Systems

Satellite Events
4 Tutorials, 21 Workshops

Invited Speakers
Rajeev Alur, Pennsylvania
Jean-Marc Eber, Paris
Stephen Gilmore, Edinburgh
Steven Miller, Rockwell Collins
John Reynolds, Carnegie Mellon
Vivek Sarkar, Rice
Wolfgang Thomas, Aachen



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Le Menu

Le Menu

Saturation Séquentièlle

A taste of symbolic state-space generation

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Saturation Parallèle

2 Cilk-flavoured implementations

1 Pthreads-flavoured implementation

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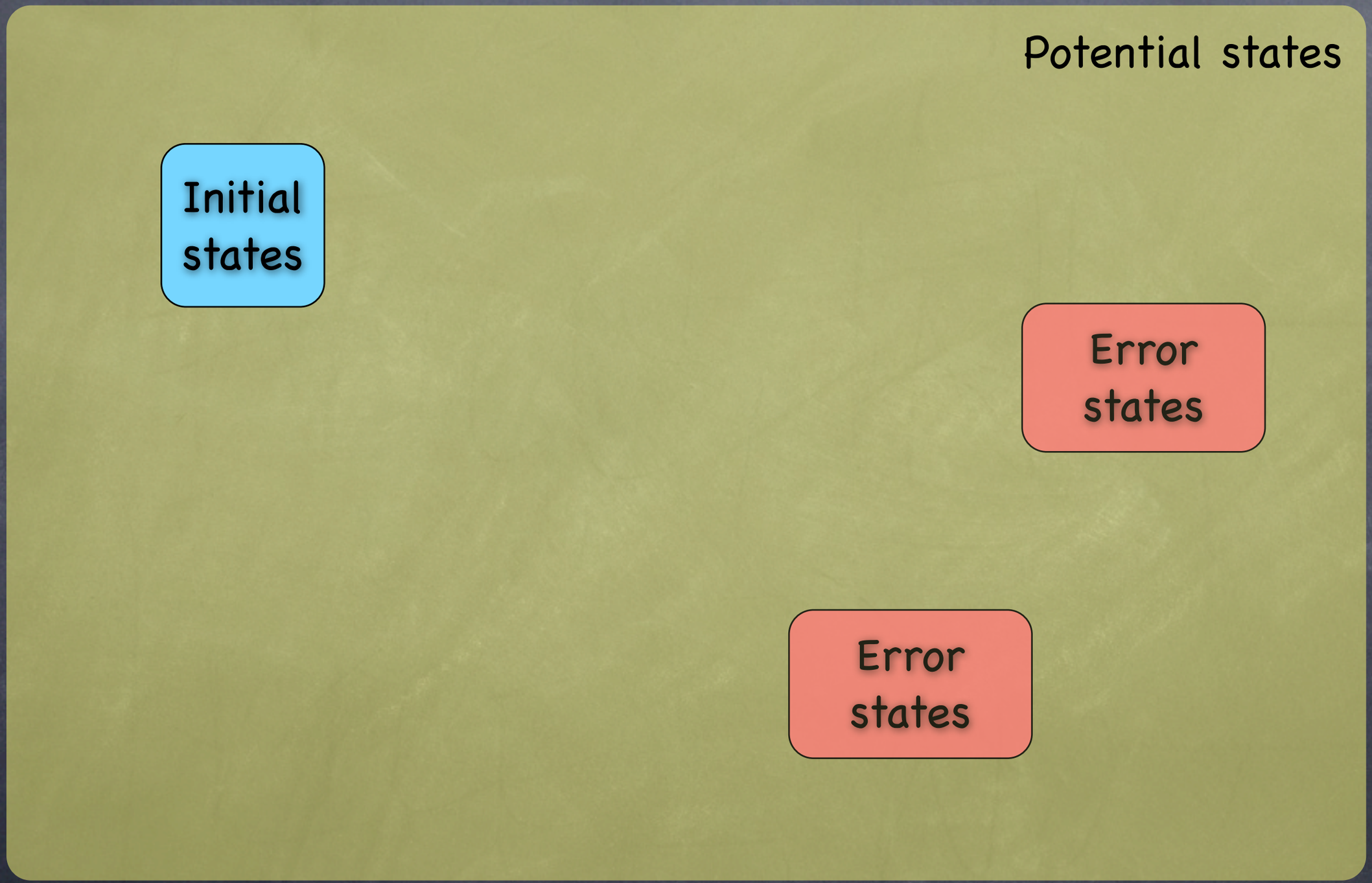
A crunch of experimental results

Frustrations and hopes: Can you do soufflé?

Conclusions: What have we learned?

Symbolic Reachability Analysis



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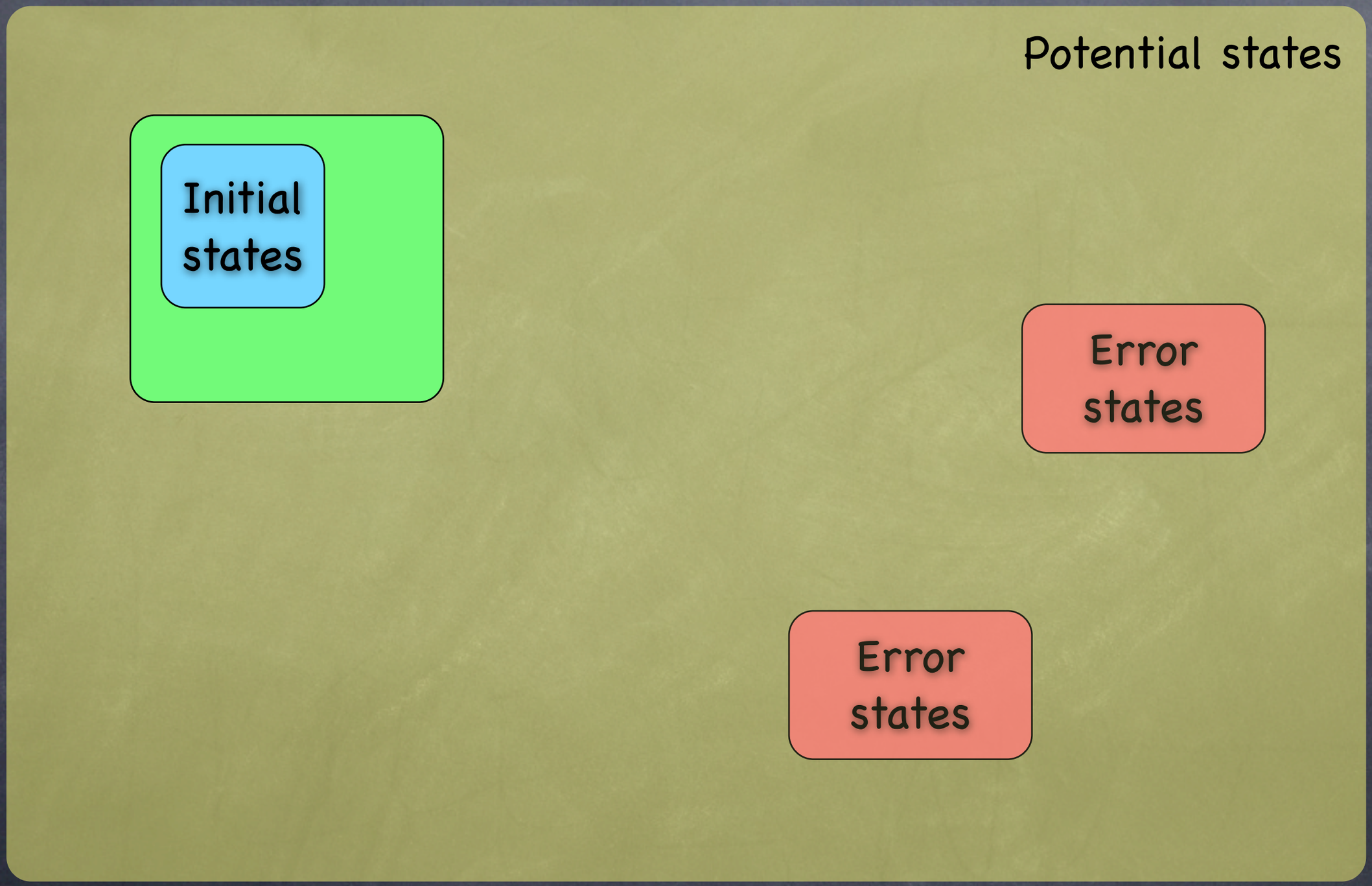

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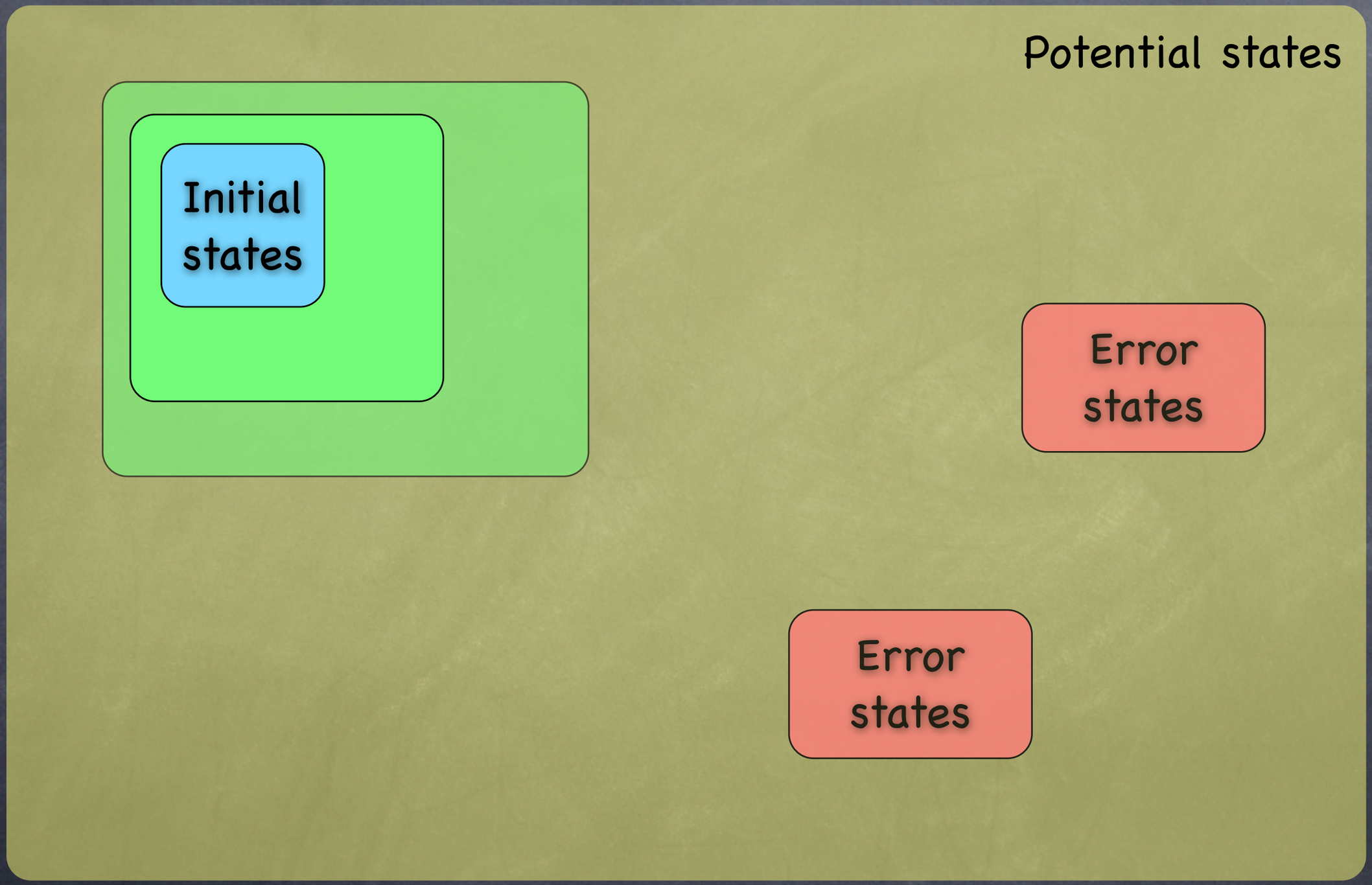

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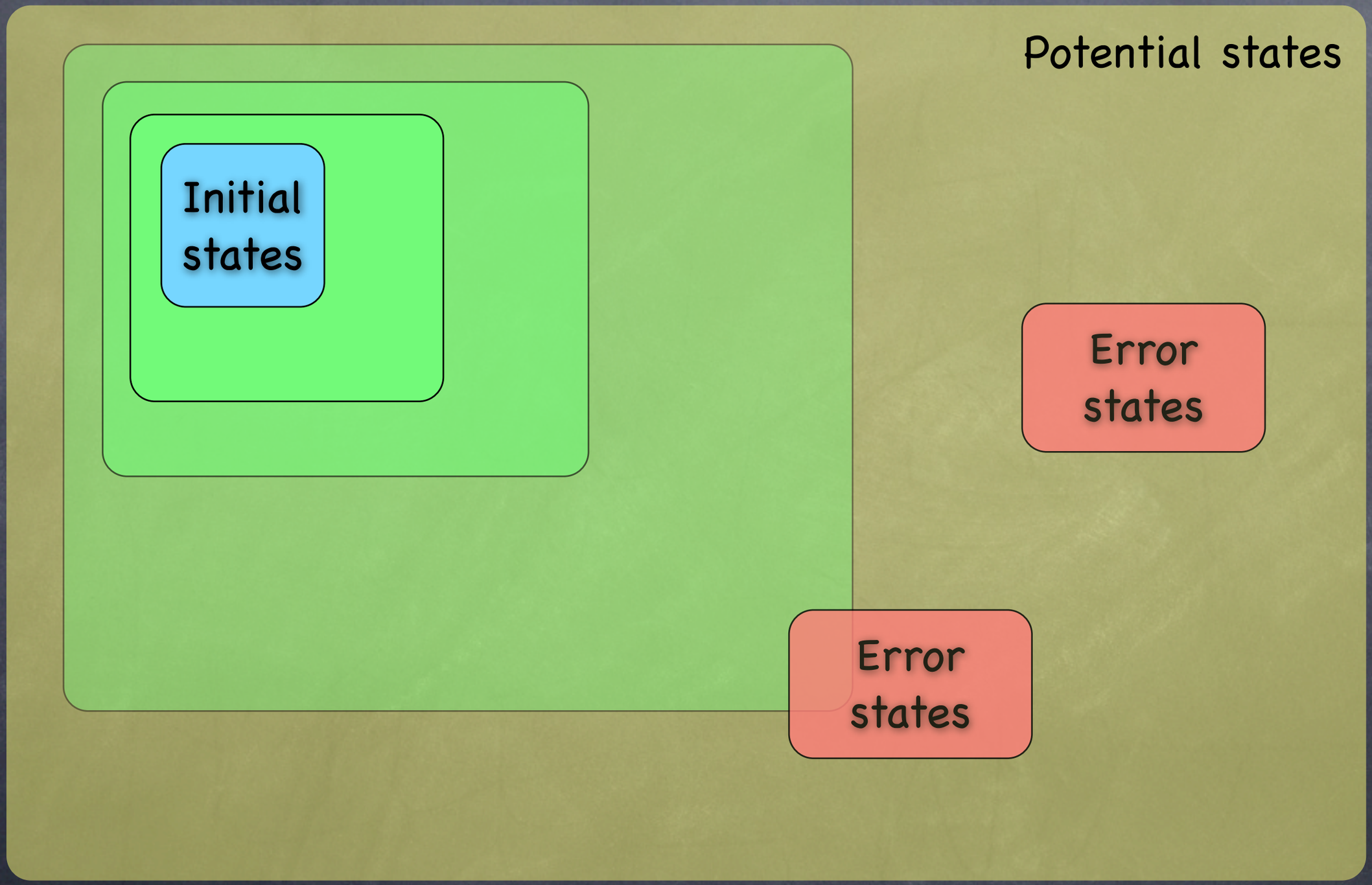

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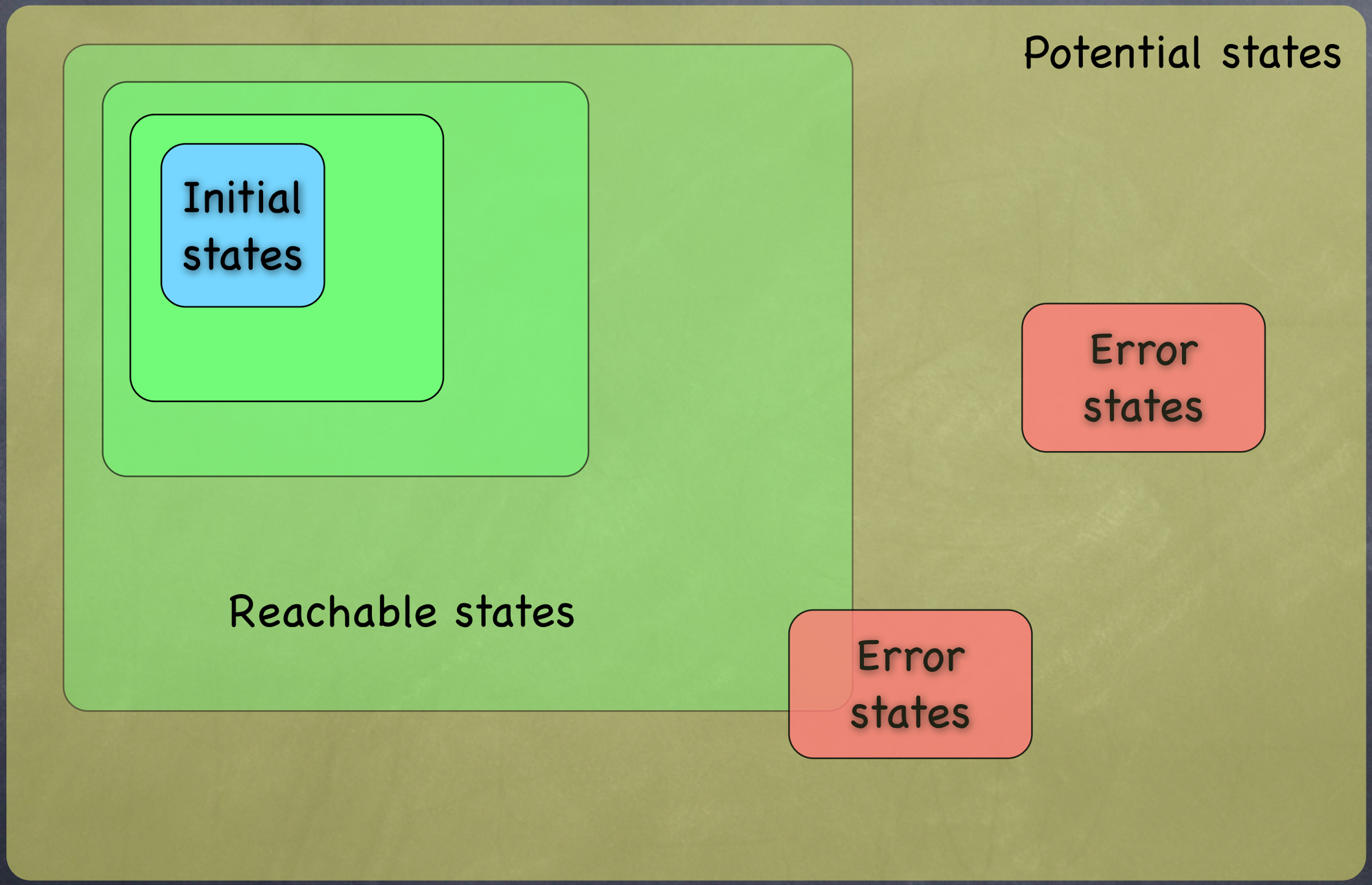

State Spaces as Fixed Points

State Spaces as Fixed Points

- What is a system/model?
  - A set of state variables $X = \{x_K,\ldots,x_1\}$, with the set States of potential states being the set of assignments over X
  - A set of initial states S_{init}
  - A next-state function $N : States \rightarrow 2^{States}$

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  - A next-state function $N : States \rightarrow 2^{States}$
- The reachable state space is the result of a least fixed point computation
  - $S_0 = S_{init}$
  - $S_{i+1} = Union(S_i, N(S_i))$

Asynchronous Models & Breadth-First Search

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- Asynchronous models are typically event-based
  - A set Events of events with representative e
  - A disjunctively partitioned next-state function

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- BFS-based state-space generation with chaining

```
S = Sinit
do
    for each e∈Events do S = Union(S, Ne(S))
while S does not change
```

Saturation: A Better Search Strategy

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- Asynchronous systems exhibit event locality
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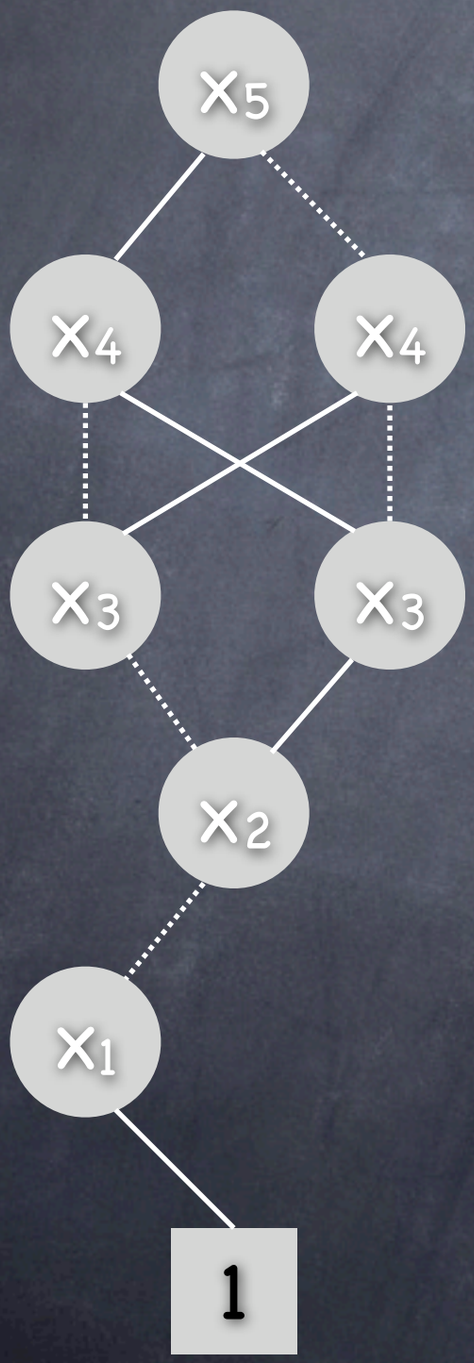
- Saturation, explained via an example model having
  - A state vector of 5 variables: x_5, x_4, x_3, x_2 and x_1
  - Three events e_{21}, e_2 and e_1 which affect variables x_2 and x_1, only variable x_2 and only variable x_1, respectively

Decision Diagrams + Partitioning by Event

State space



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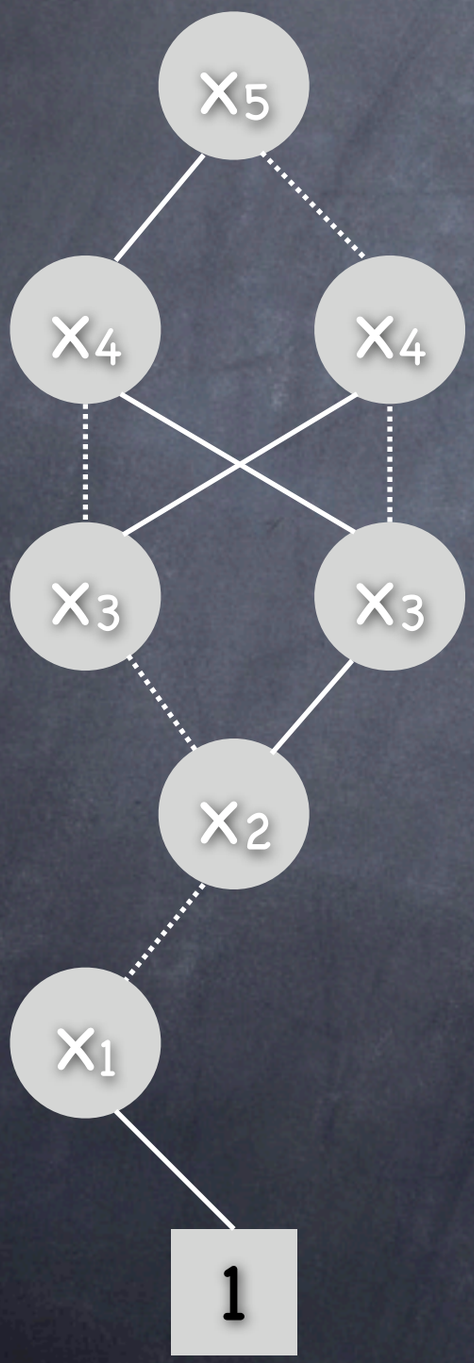

Next-state function

$$N(X) = U_{e \in Events}\ N_e(X)$$

$$= N_{e21}(X) \cup N_{e2}(X) \cup N_{e1}(X)$$

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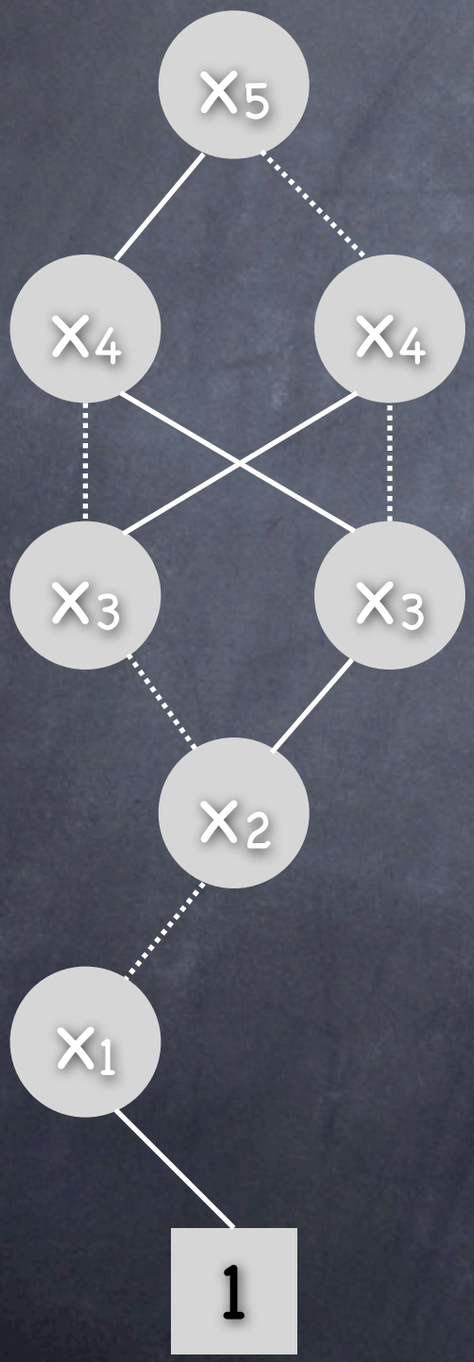

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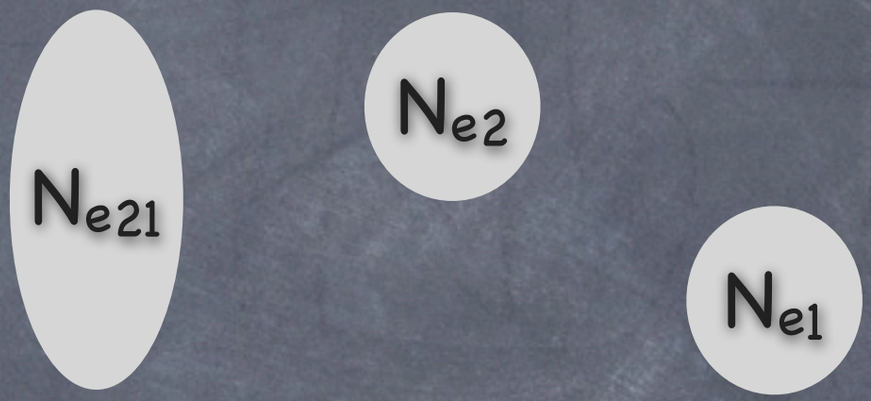
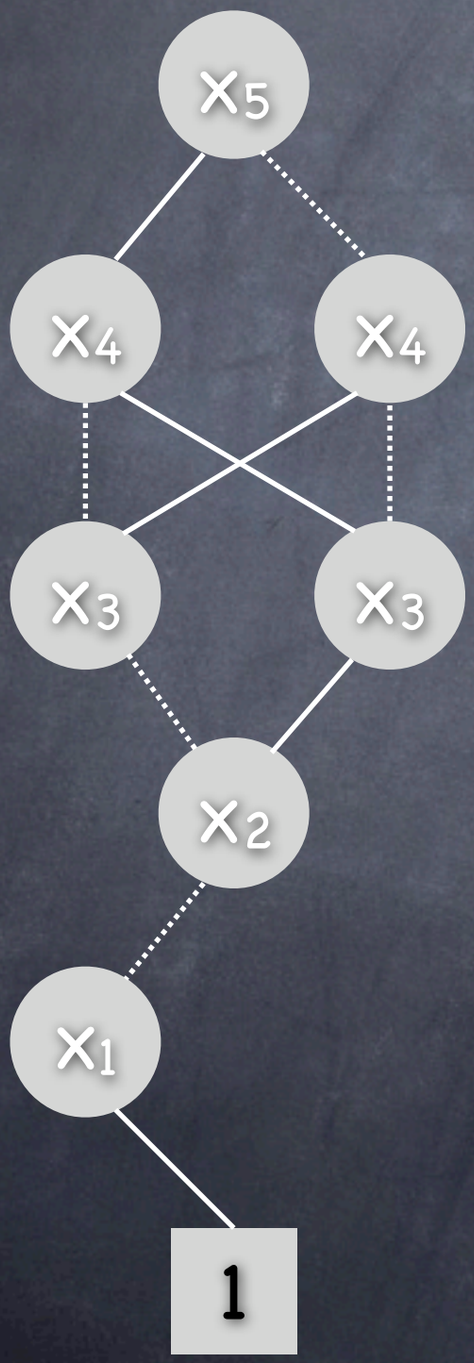


In the symbolic encoding of decision diagrams, each event is assigned a top-level: $Top(e_{21}) = 2$, $Top(e_2) = 2$ and $Top(e_1) = 1$

Partitioning by Top + Identity Reduction

State space



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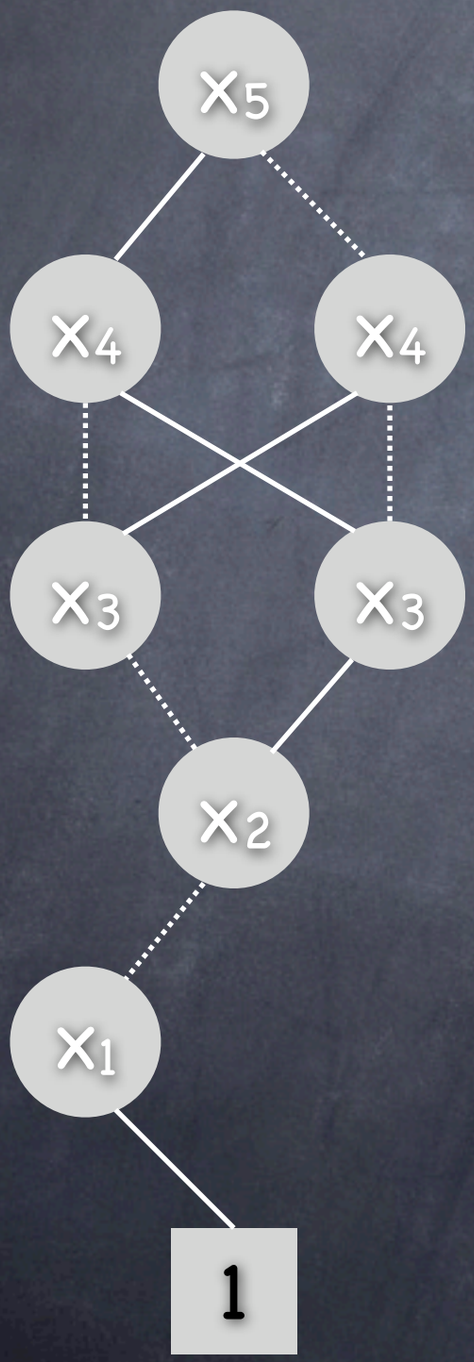

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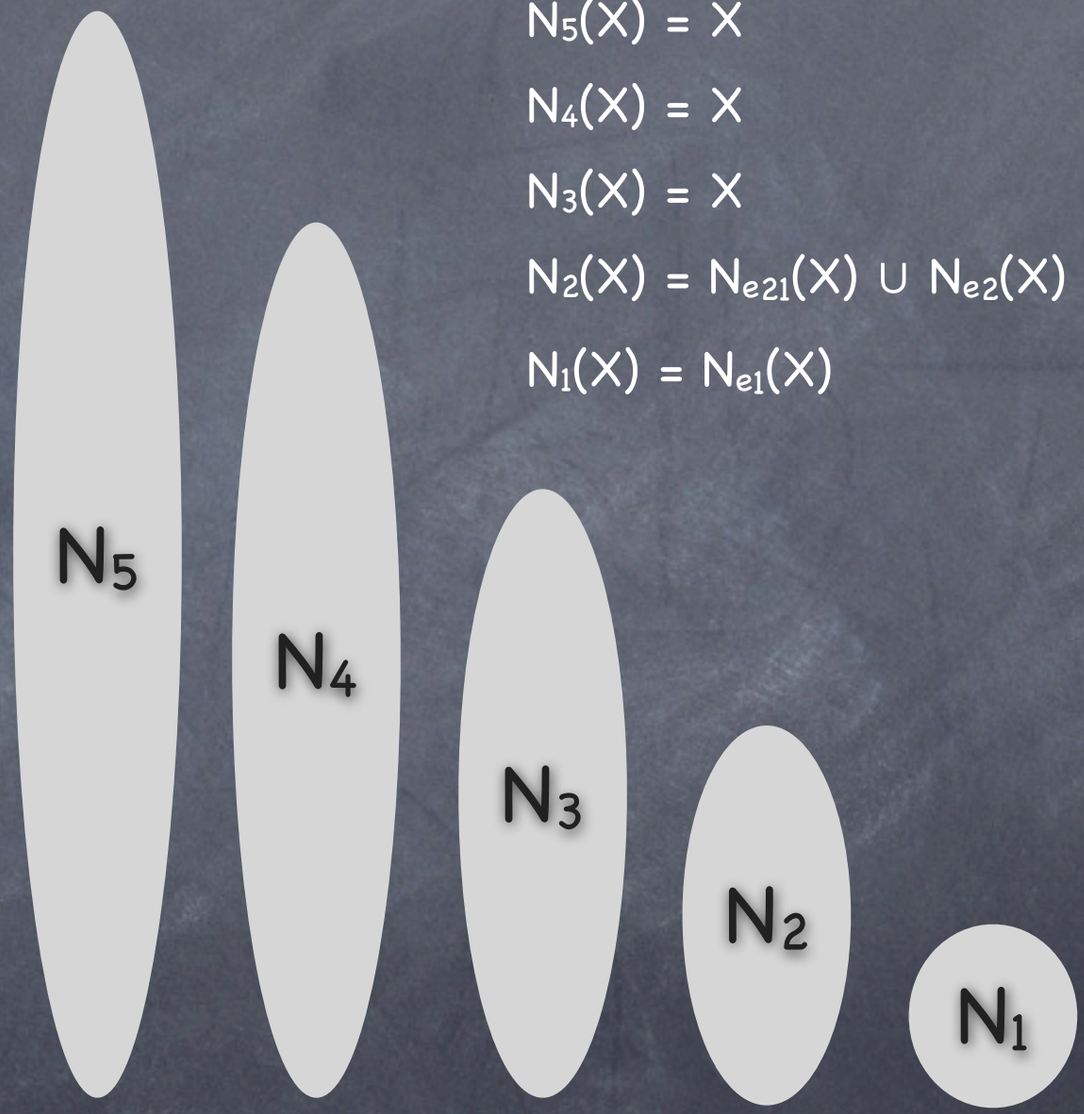

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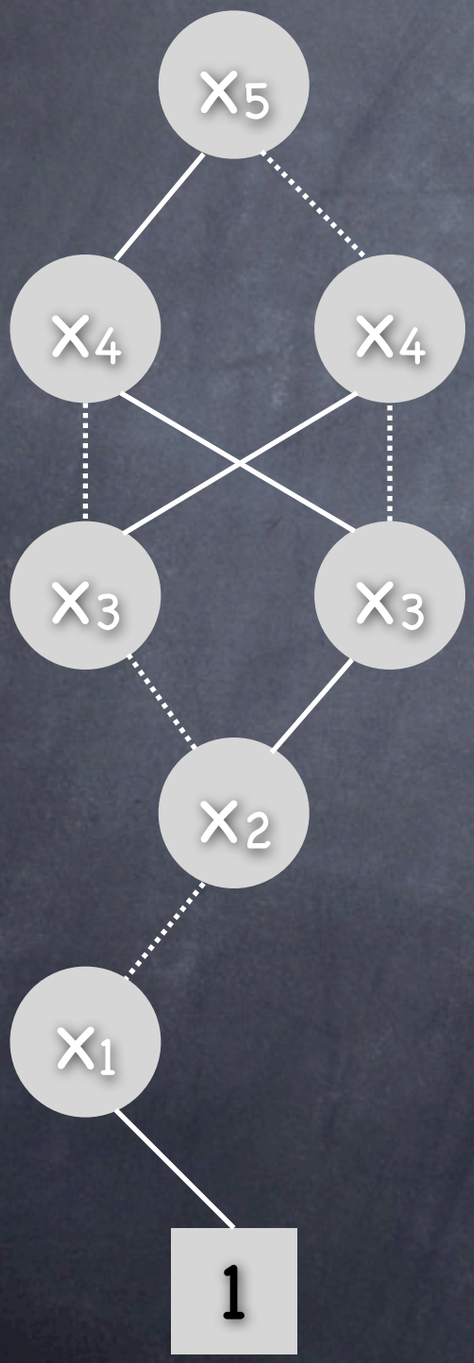

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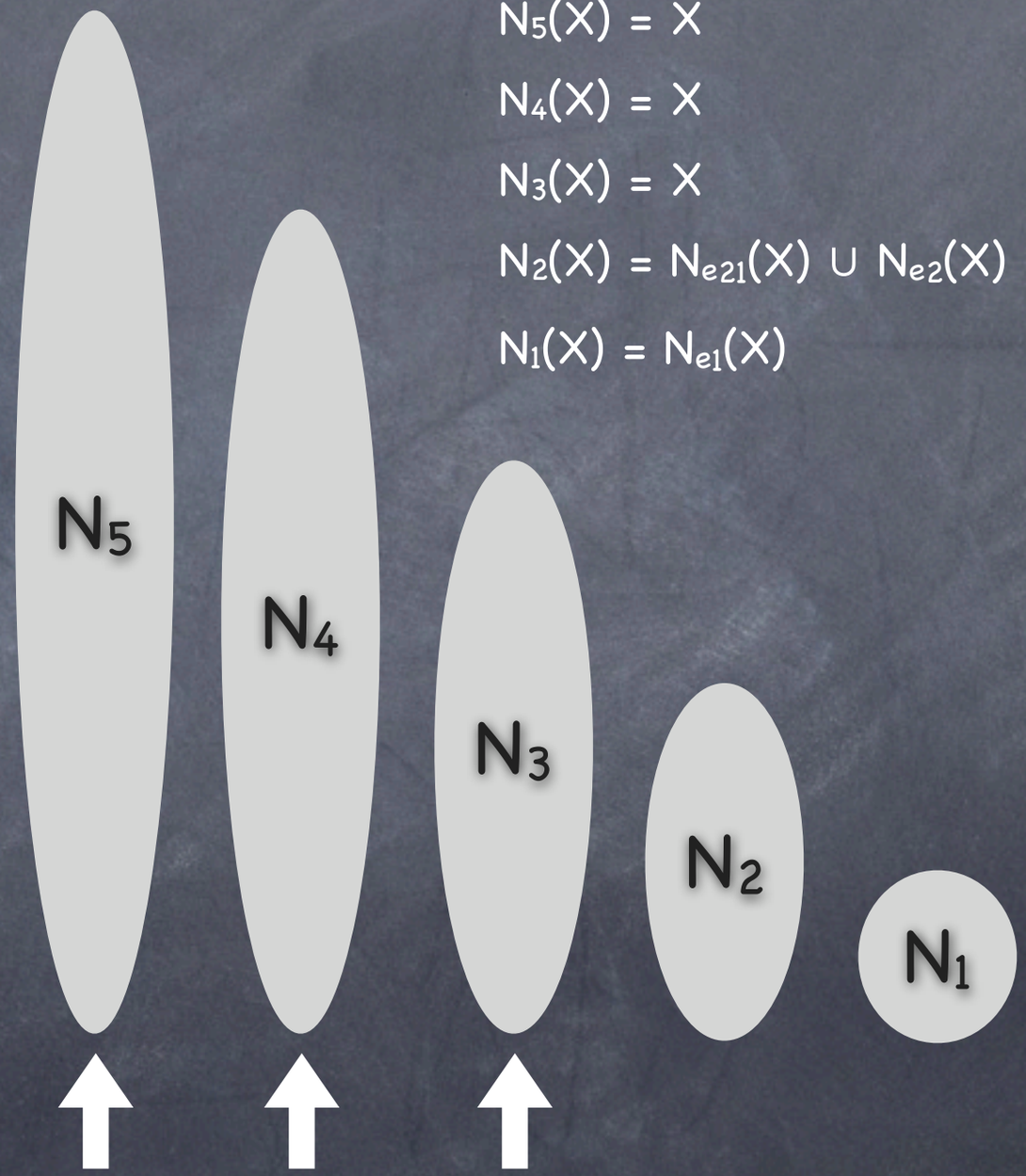
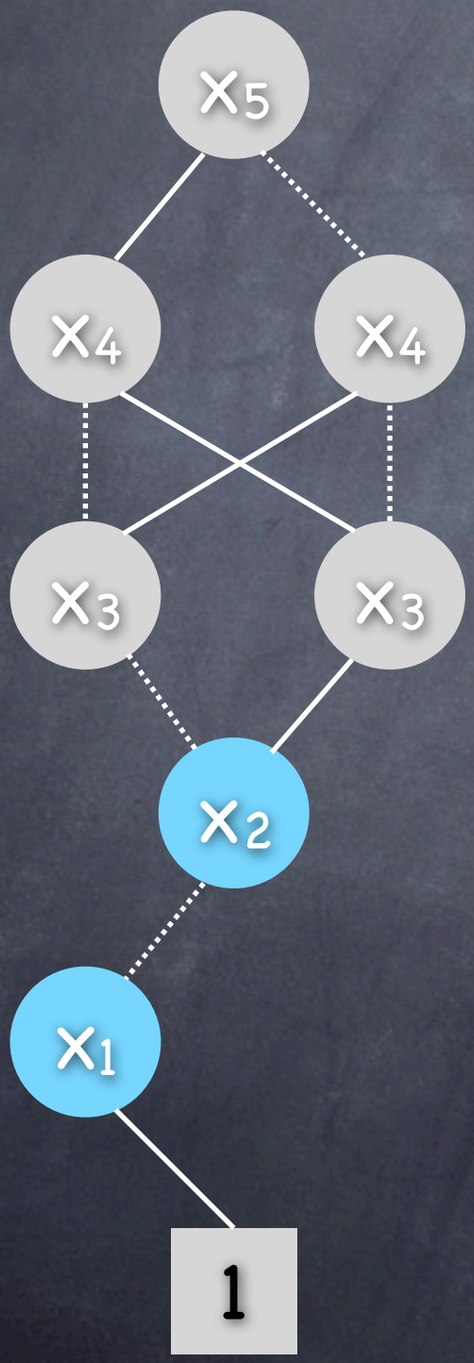


Identity transformations are skipped

Standard BFS-based Image Computation

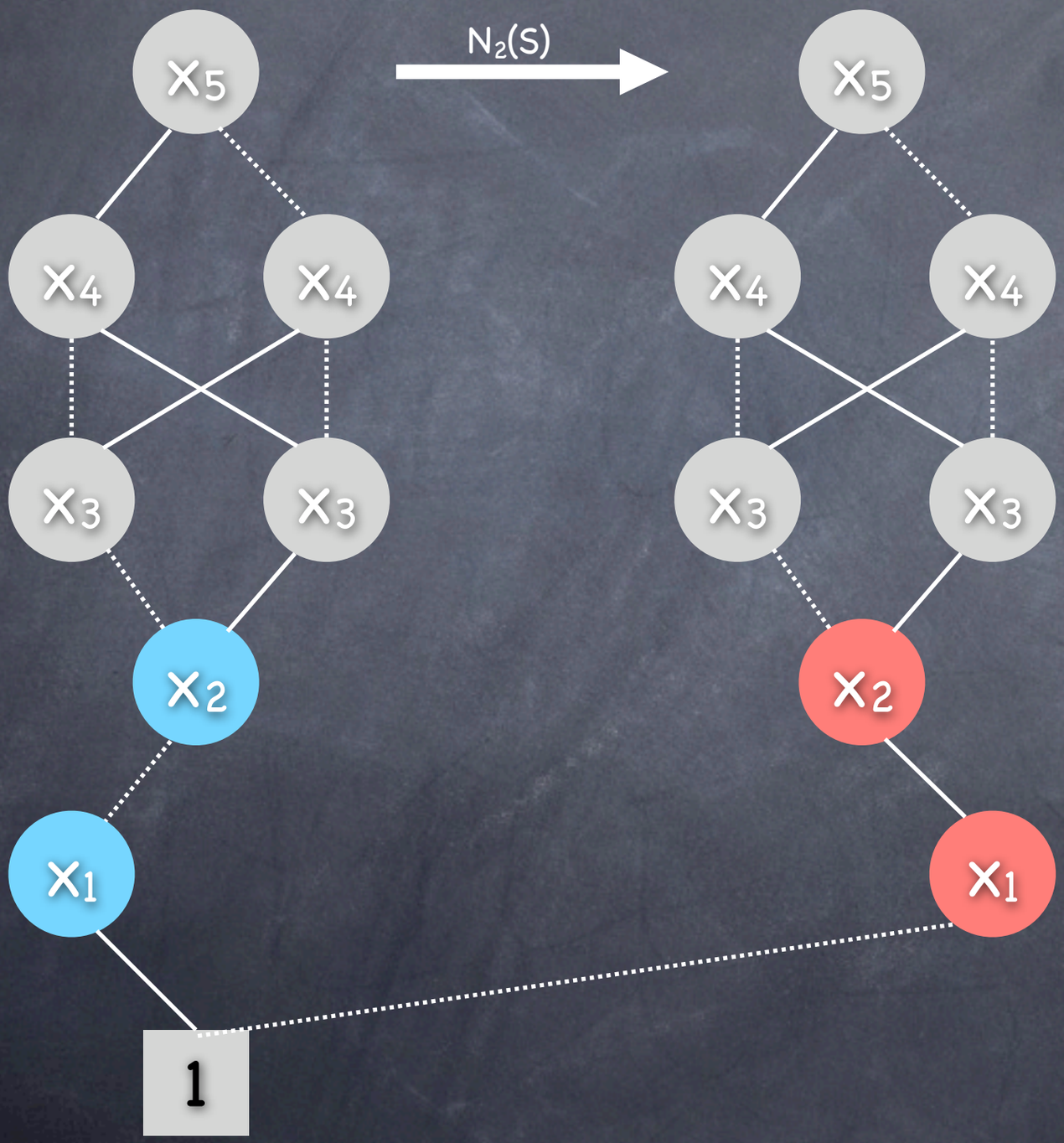
Current states S



Standard BFS-based Image Computation

Current states S

New image



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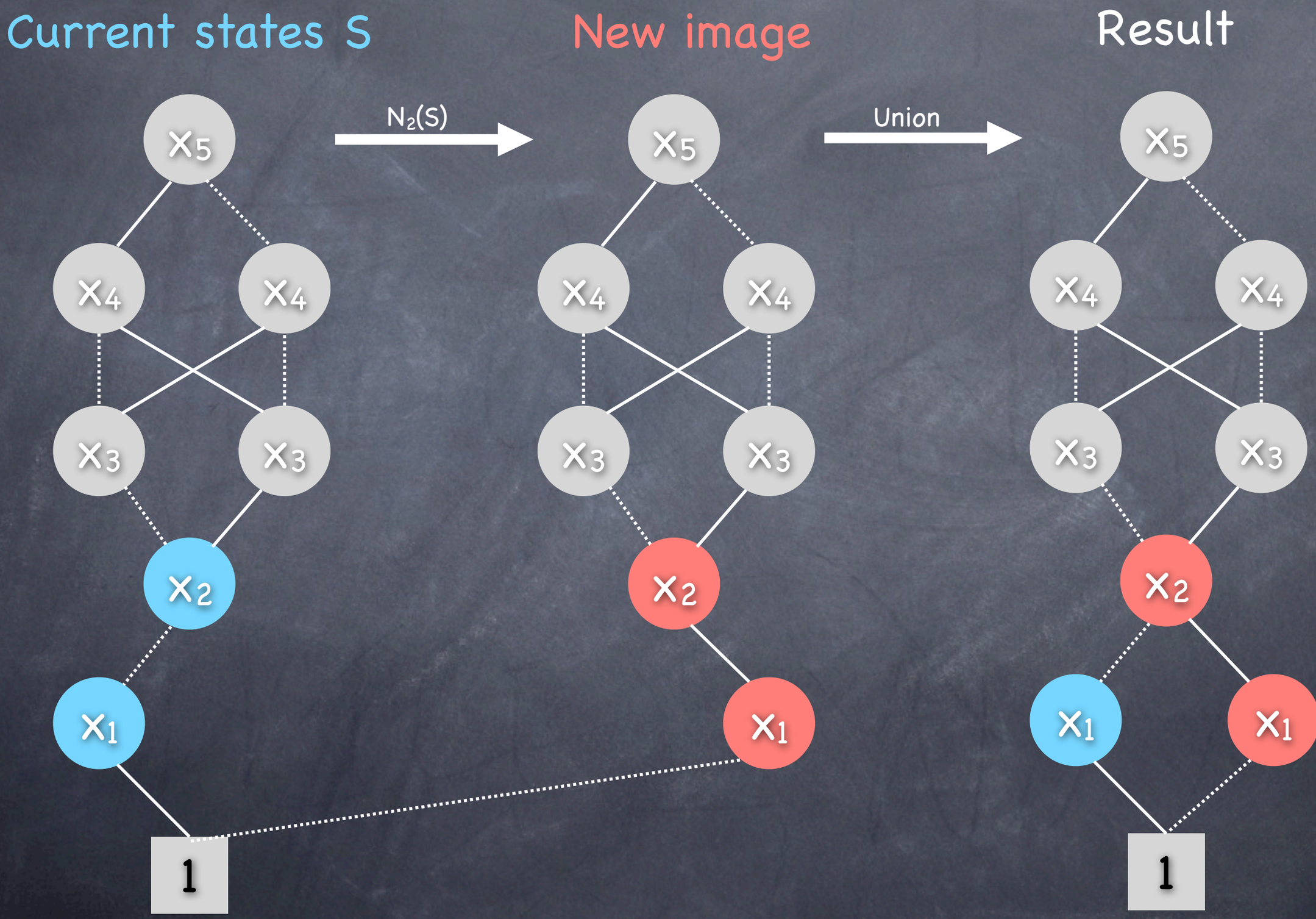

Exploiting Event Locality in Image Computation

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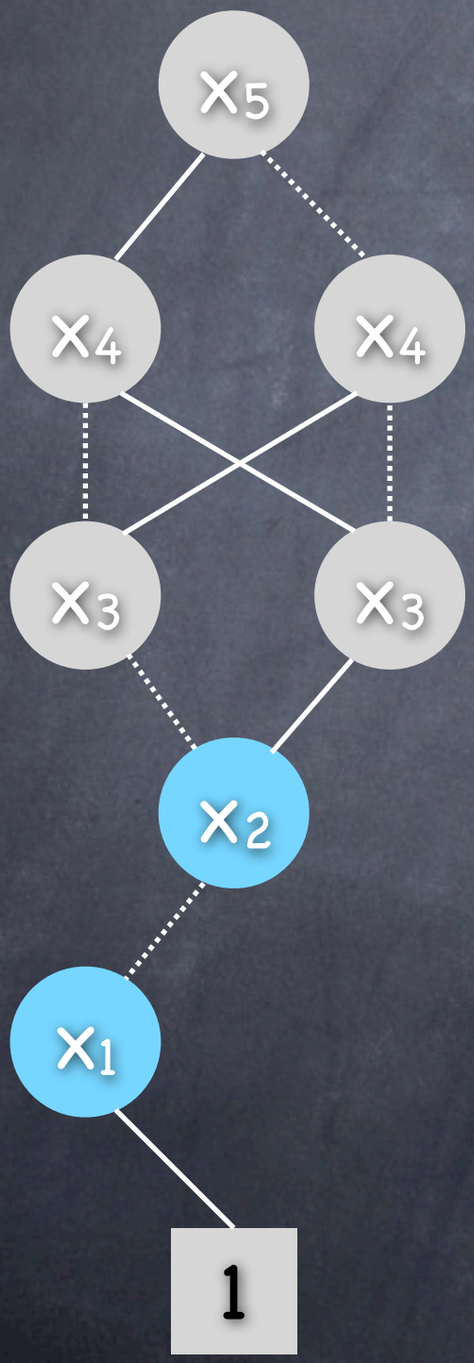

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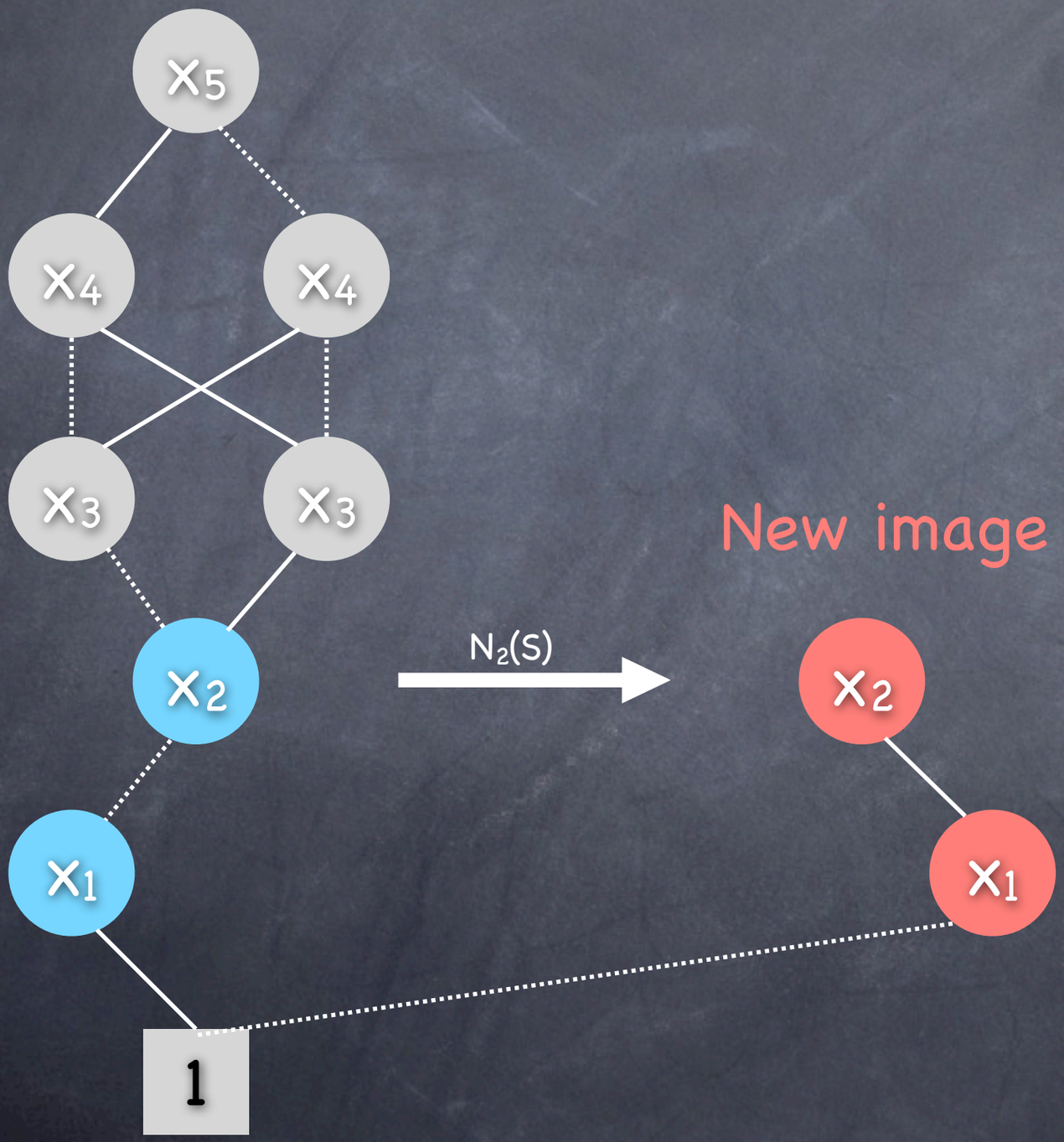

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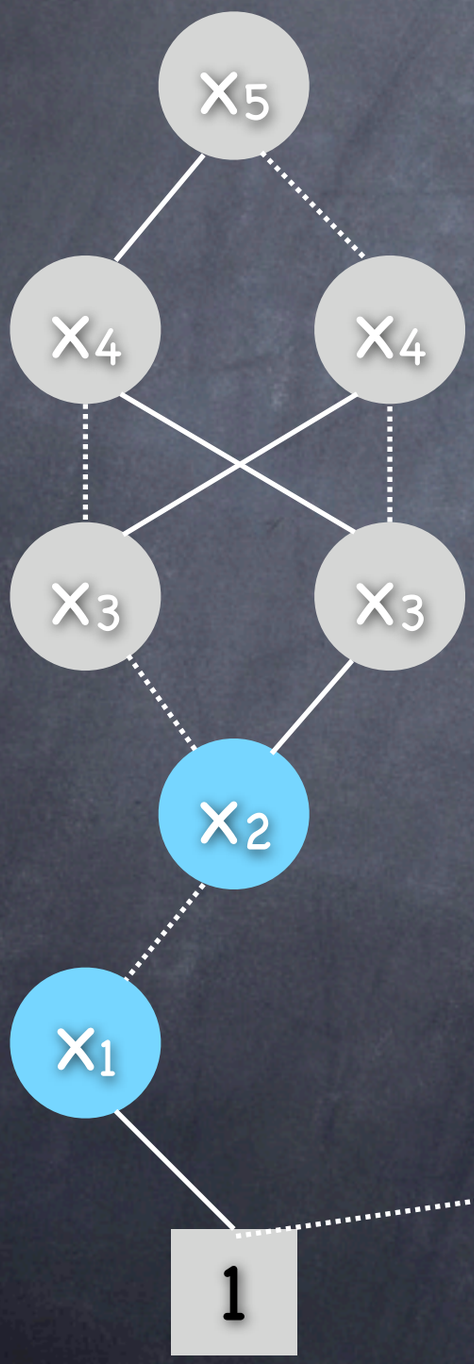
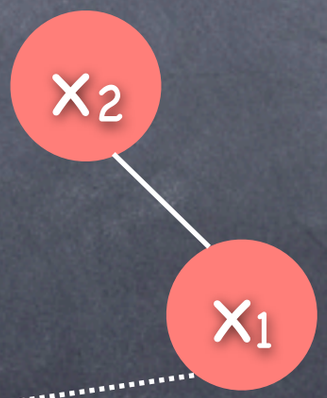


New image

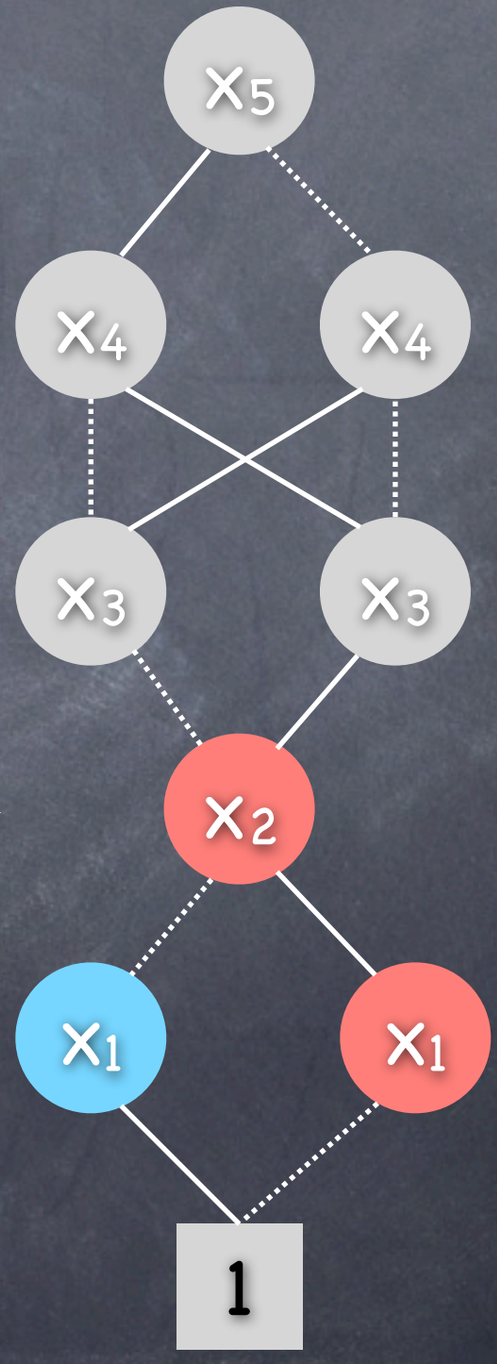
$N_2(S)$



Union +

In-place update

Result



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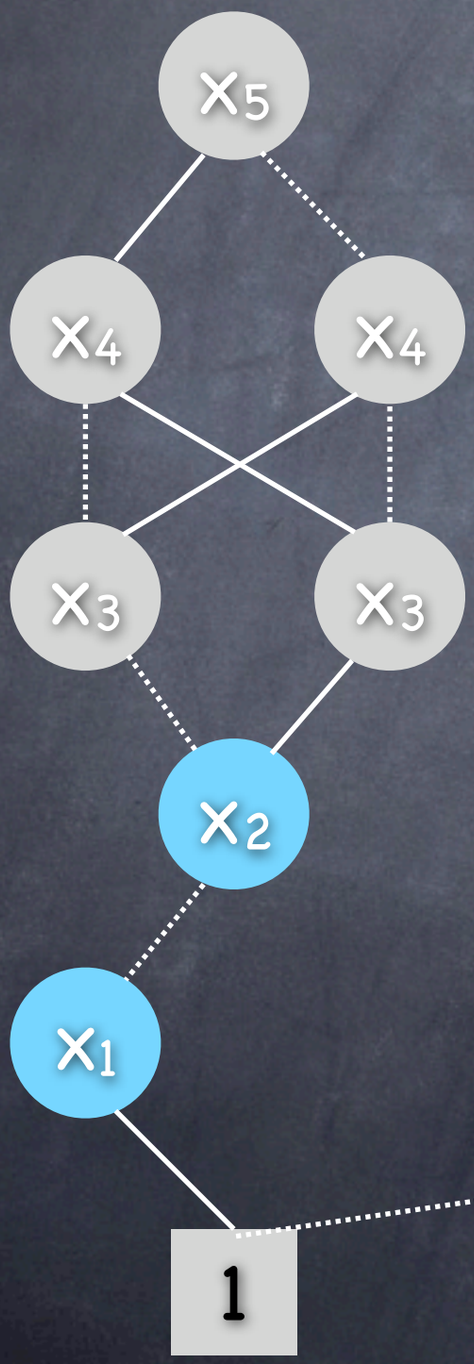
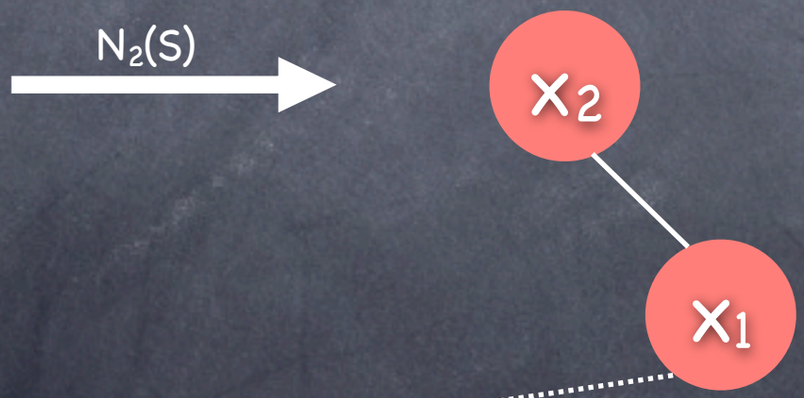


Reduces peak number of nodes by a large factor!

New image



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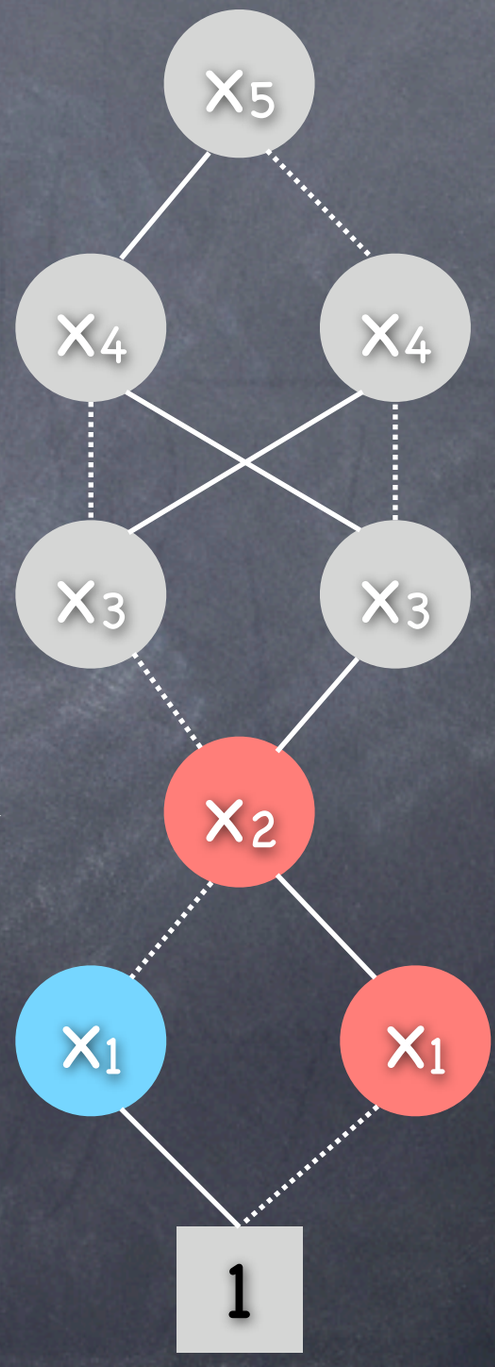
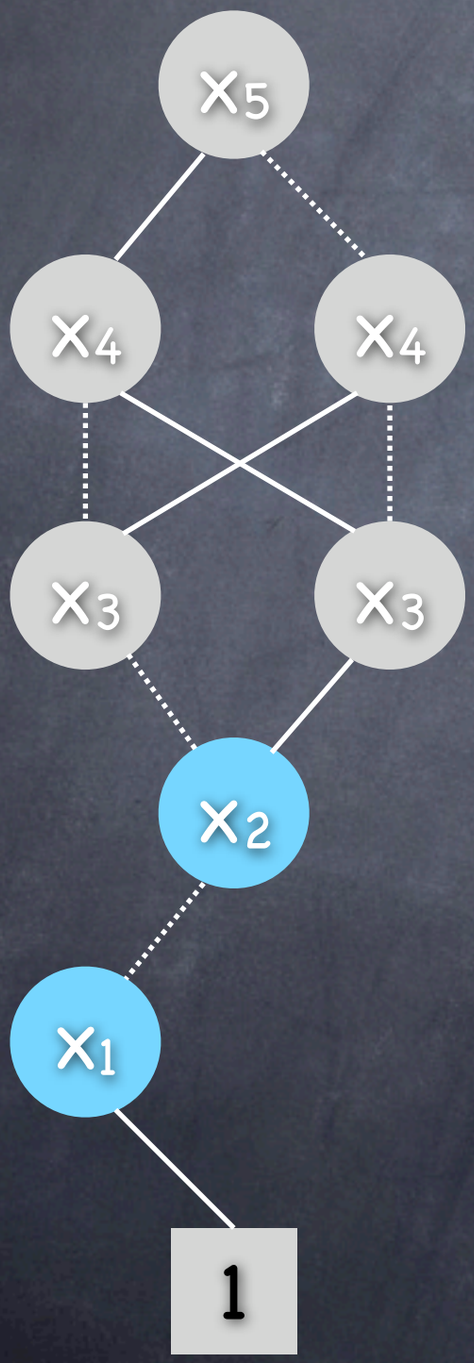


Saturation: Computing Fixed Point at Each Node

Current states



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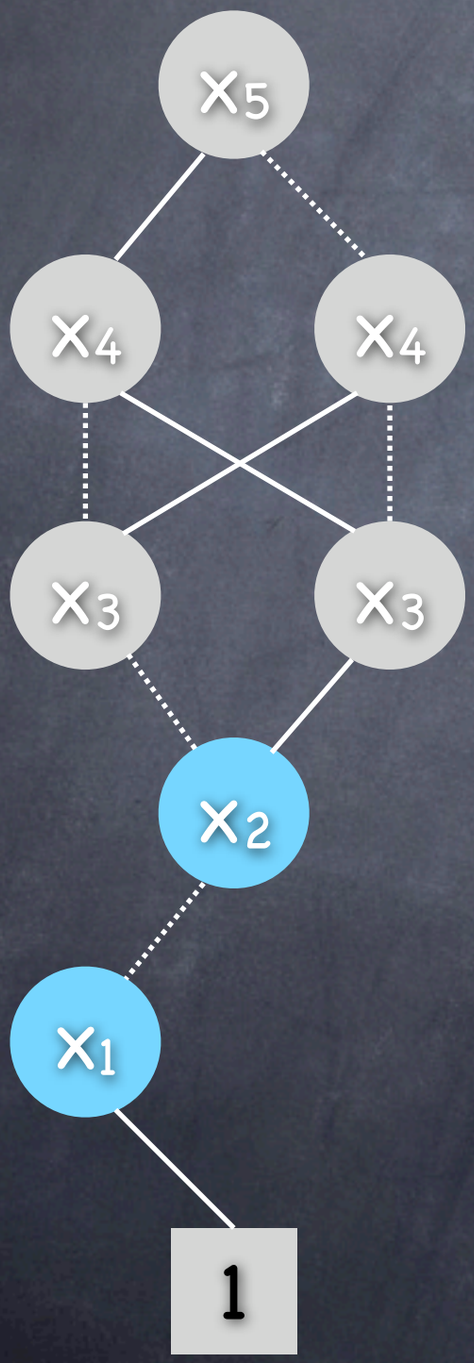
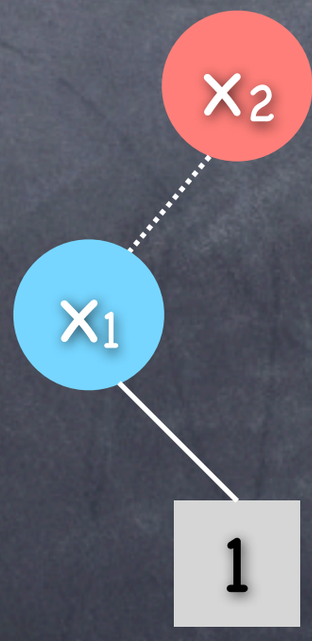


Fire e_{21} + e_2 repeatedly +

Saturate new node

fixed point e_1



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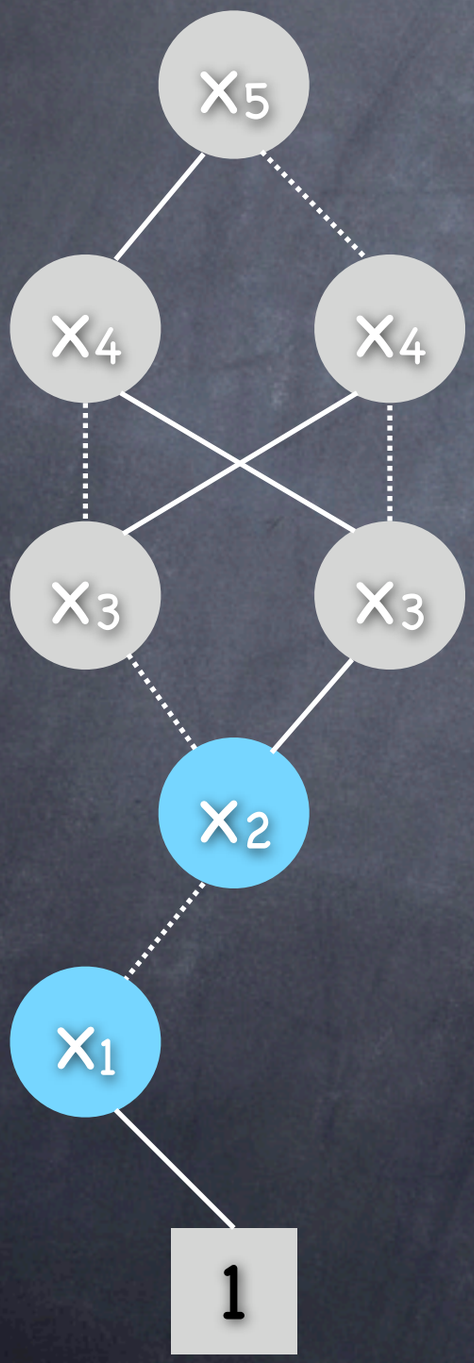

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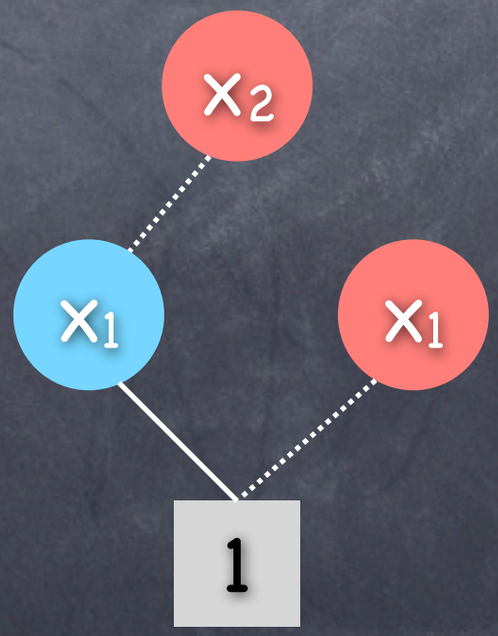

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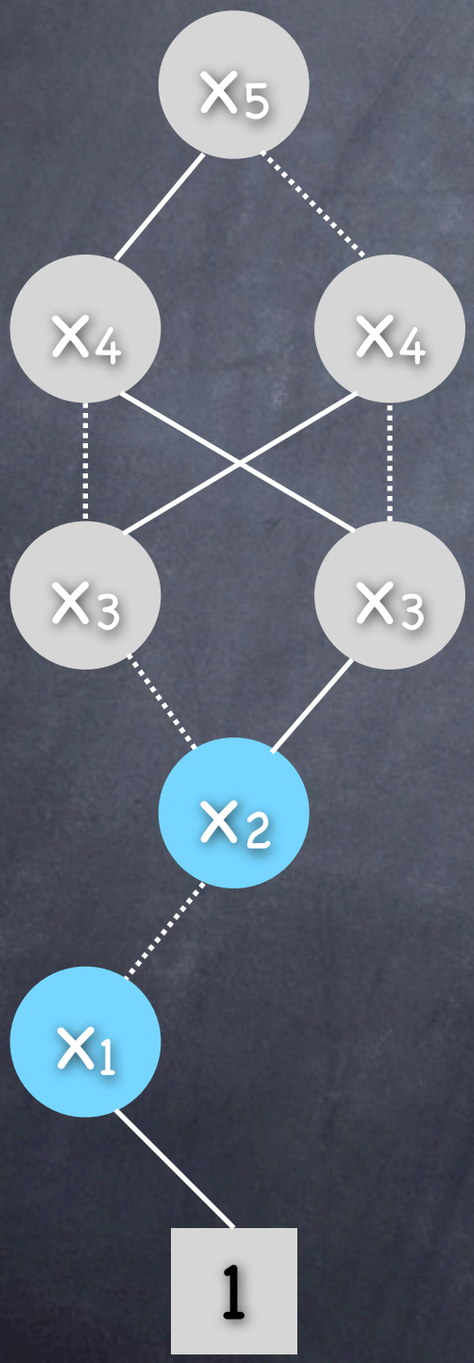

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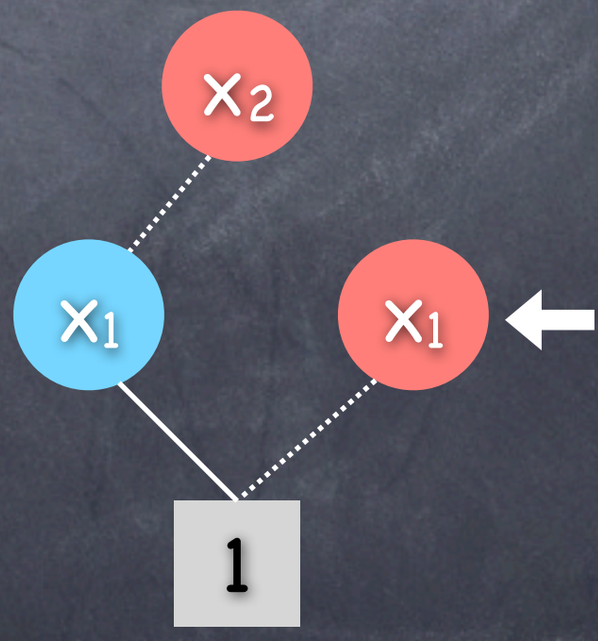

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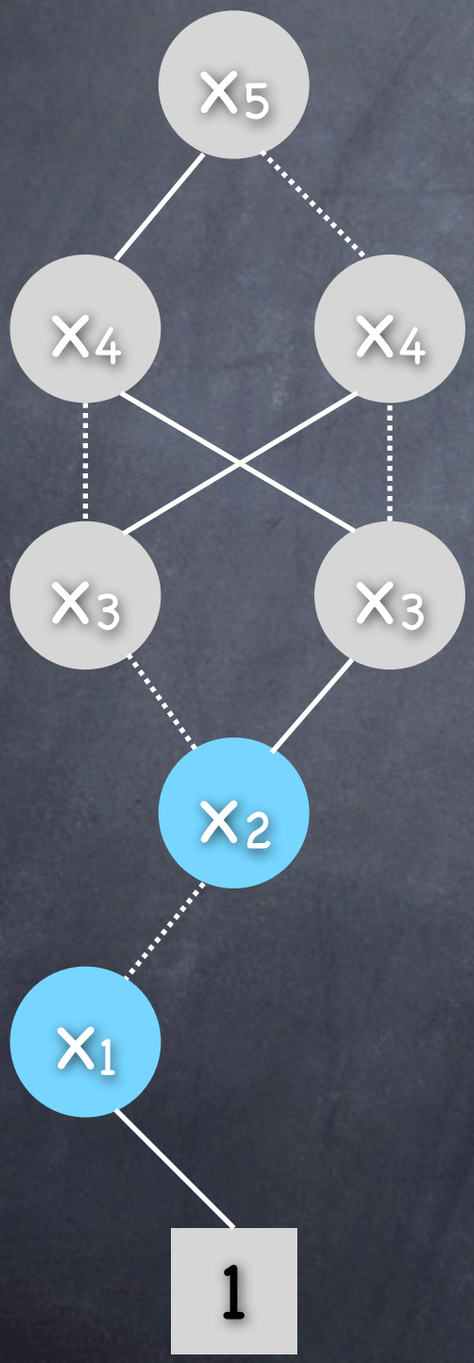

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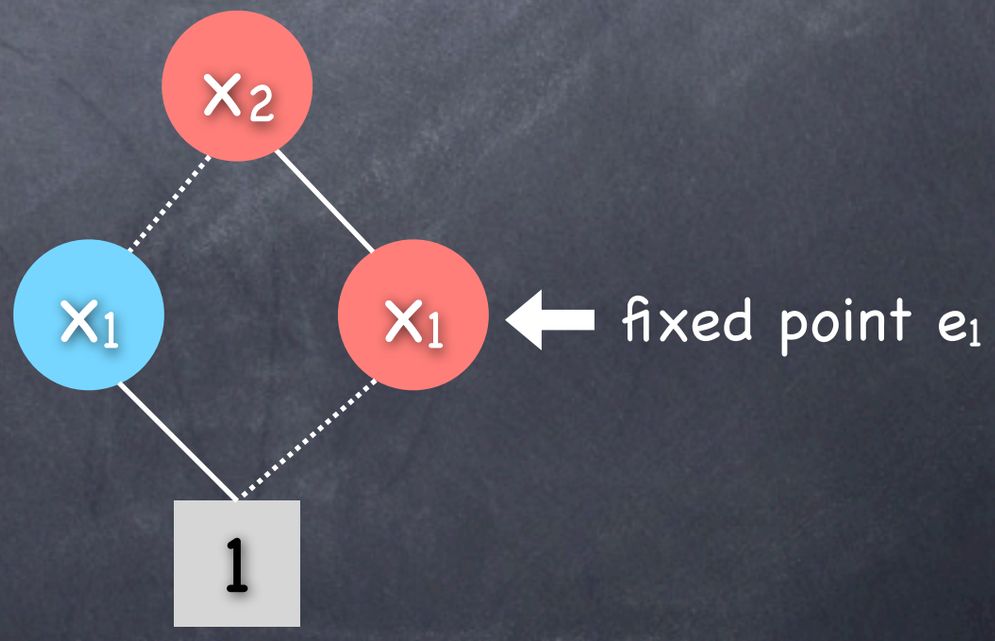

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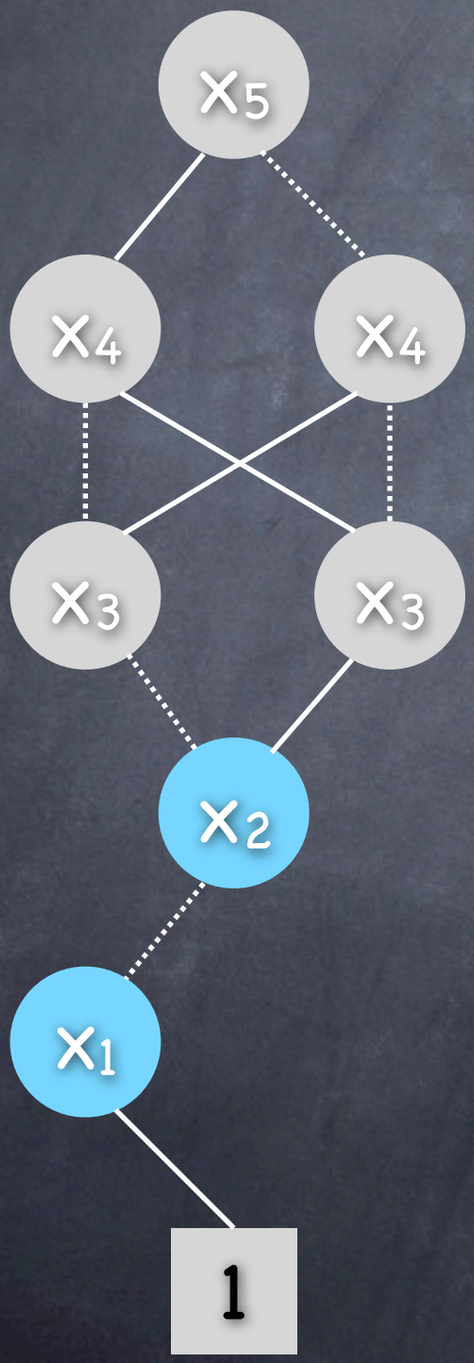

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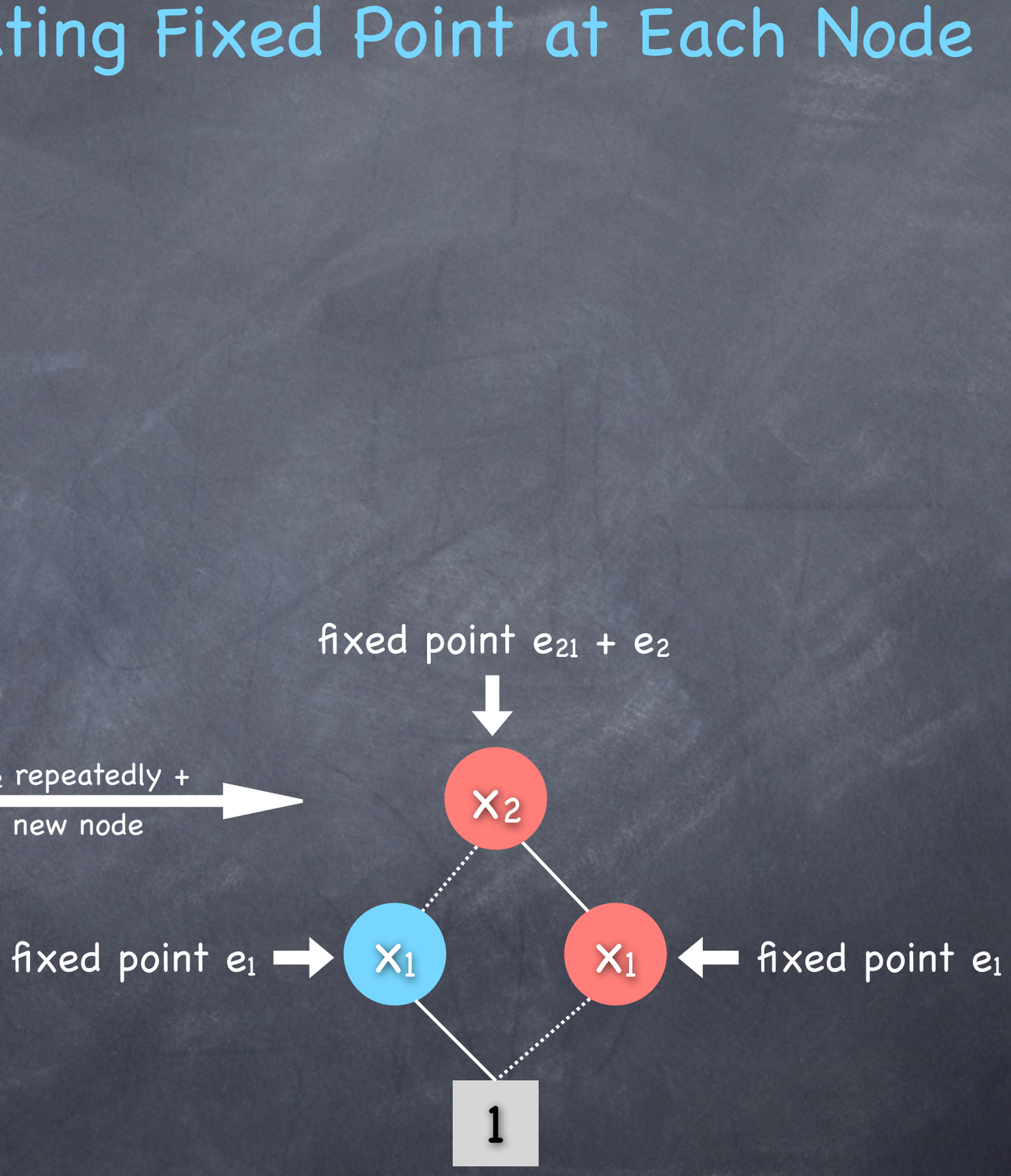

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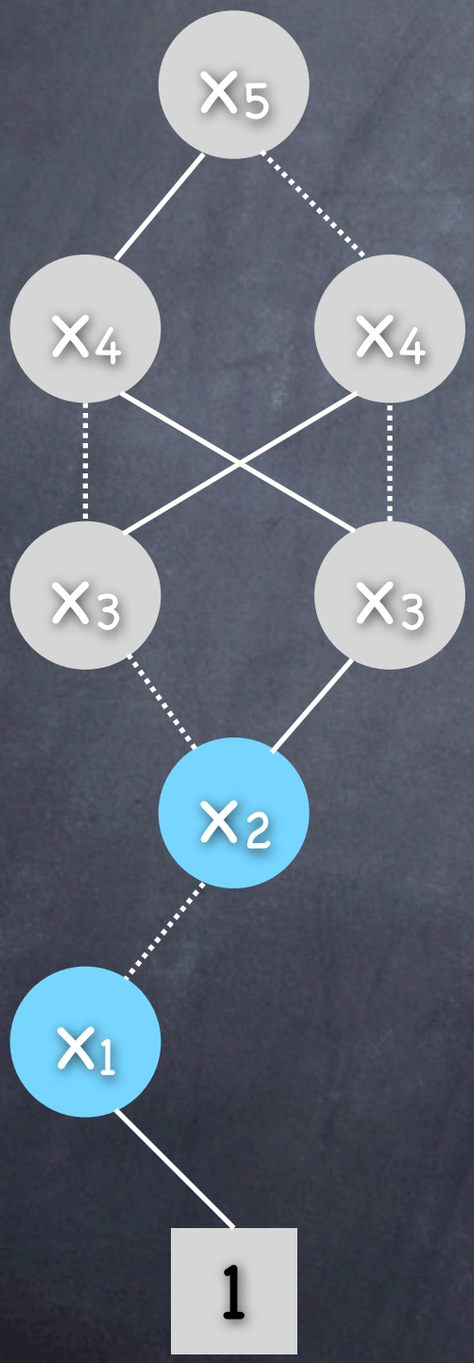
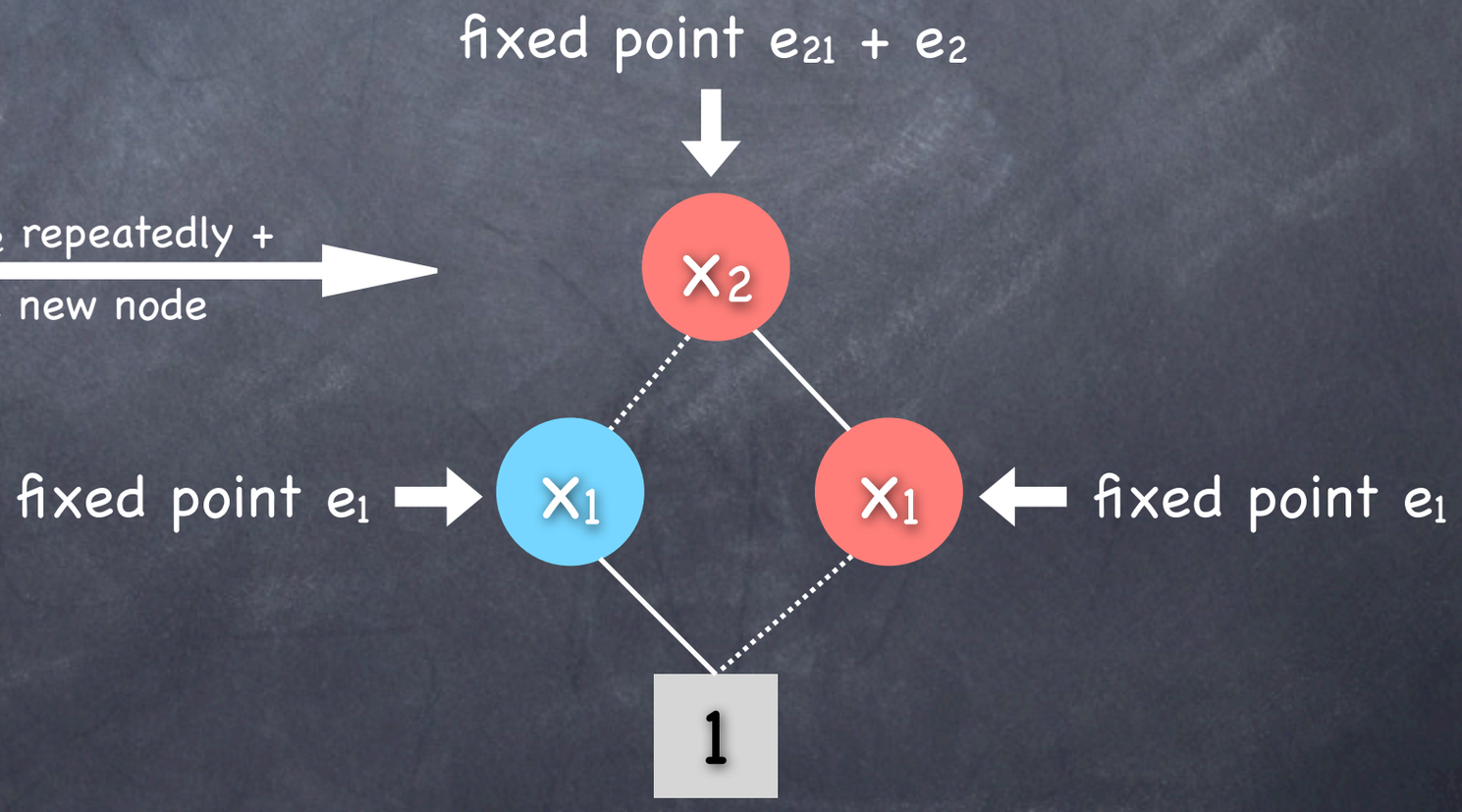


Reduces run-time and memory usage by several orders of magnitude!

Fire $e_{21} + e_2$ repeatedly + Saturate new node



Parallelising Saturation

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- Idea:
  - When saturating a node, fire several events simultaneously (including the same event enabled in several local states)
- Two implementation strategies:
  - Use a dedicated parallel language, such as Cilk (MIT)
  - Write parallelisation by hand, using Pthreads (POSIX threads)

Parallelising Saturation

- ... for shared-memory machines such as multi-core PCs
- Idea:
  - When saturating a node, fire several events simultaneously (including the same event enabled in several local states)
- Two implementation strategies:
  - Use a dedicated parallel language, such as Cilk (MIT)
  - Write parallelisation by hand, using Pthreads (POSIX threads)
- Questions:
  - Does it work? Is it worthwhile?

Parallelising Saturation on Multi-Core PCs

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- Symbolic state-space generation is irregular in nature
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➡ Load balancing becomes a major issue!

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- Operating system threads are not an option
  - Thread creation ~ 12,000 ns; thread allocation ~ 3,000 ns

Related Work - A Rough Overview

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- Most research on symbolic state exploration parallelises decision diagrams on PC clusters
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- Everyone parallelises "by hand", mostly using MPI or Pthreads; no-one employs a dedicated parallel language!

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- Is aimed at divide-and-conquer problems rather than producer-consumer problems

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- Spawning a Cilk function by using the keyword spawn when calling the function
  - Multiple functions can be spawned within a calling function
  - Calling function continues to execute in parallel
- Synchronising spawned threads by using the keyword sync, which prevents the calling function to continue until all of its spawned functions have completed
  - Cilk functions contain an implicit sync before they are allowed to return

More Cilk Language Constructs

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- Handling return values either left implicit or to an inlet
  - An inlet is a function local to a Cilk function, which handles the result of a spawned function
  - Only one completed Cilk function can be handled at a time by an inlet, i.e., inlets are atomic
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  - Inlets may not use further spawn or sync statements, i.e., no expression of pipelining or producer-consumer problems
- Specifying explicit mutex locks is supported in Cilk
  - So we have control over locking our hash tables and caches

Parallel Saturation in Cilk: First Variant

cilk $Saturate$(in k:lvl, p:$node$)

Update p, a node at level k not in the hash table, in–place, to encode $\mathcal{N}^*_{\leq k}(\mathcal{B}(p))$.

declare $pCng : bool$; $e : event$; $i, j : lcl$;
declare $\mathcal{L}$: set of lcl; $u : node$;

1. inlet void $DoUnion(f : lcl)$ {
2. if $f \neq \mathbf{0}$ then
3. foreach $j \in \mathcal{N}_{k,e}(i)$ do
4. $u \Leftarrow Union(k-1, f, p[j])$;
5. if $u \neq p[j]$ then
6. $p[j] \Leftarrow u$; $pCng = true$;
7. if $\mathcal{N}_{k,e}(j) \neq 0$ then
8. $\mathcal{L} = \mathcal{L} \cup \{j\}$;
9. }
10. repeat
11. $pCng \Leftarrow$ false;
12. for each $e \in \mathcal{E}_k$ do
13. $\mathcal{L} = Locals(e,k,p)$;
14. while $\mathcal{L} \neq \emptyset$ do
15. $i = Pick(\mathcal{L})$;
16. $DoUnion$(spawn $Fire(e, k-1, p[i])$);
17. sync;
18. until $pCng = false$;

cilk $Fire$(in e:$event$, l:lvl, q:$node$):$node$

Build an MDD rooted at level l, encoding $\mathcal{N}^*_{\leq l}(\mathcal{N}_e(\mathcal{B}(q)))$.

declare $\mathcal{L}$: set of lcl;
declare $i, j : lcl$;
declare $f, u, s : node$;
declare $sCng : bool$;

1. if $l < Last(e)$ then return q;
2. if $Find(FireCache[l], \{q, e\}, s)$ return s;
3. $s \Leftarrow NewNode(l)$; $sCng \Leftarrow false$;
4. $\mathcal{L} \Leftarrow Locals(e, l, q)$;
5. while $\mathcal{L} \neq \emptyset$ do
6. $i \Leftarrow Pick(\mathcal{L})$;
7. $f \Leftarrow Fire(e, l-1, q[i])$;
8. if $f \neq \mathbf{0}$ then
9. foreach $j \in \mathcal{N}_{l,e}(i)$ do
10. $u \Leftarrow Union(l-1, f, s[j])$;
11. if $u \neq s[j]$ then
12. $s[j] \Leftarrow u$; $sCng \Leftarrow true$;
13. if $sCng$ then $Saturate(l, s)$;
14. $CheckIntoHashTable(l, s)$;
15. $Insert(FireCache[l]\{q, e\}, s)$;
16. return s;

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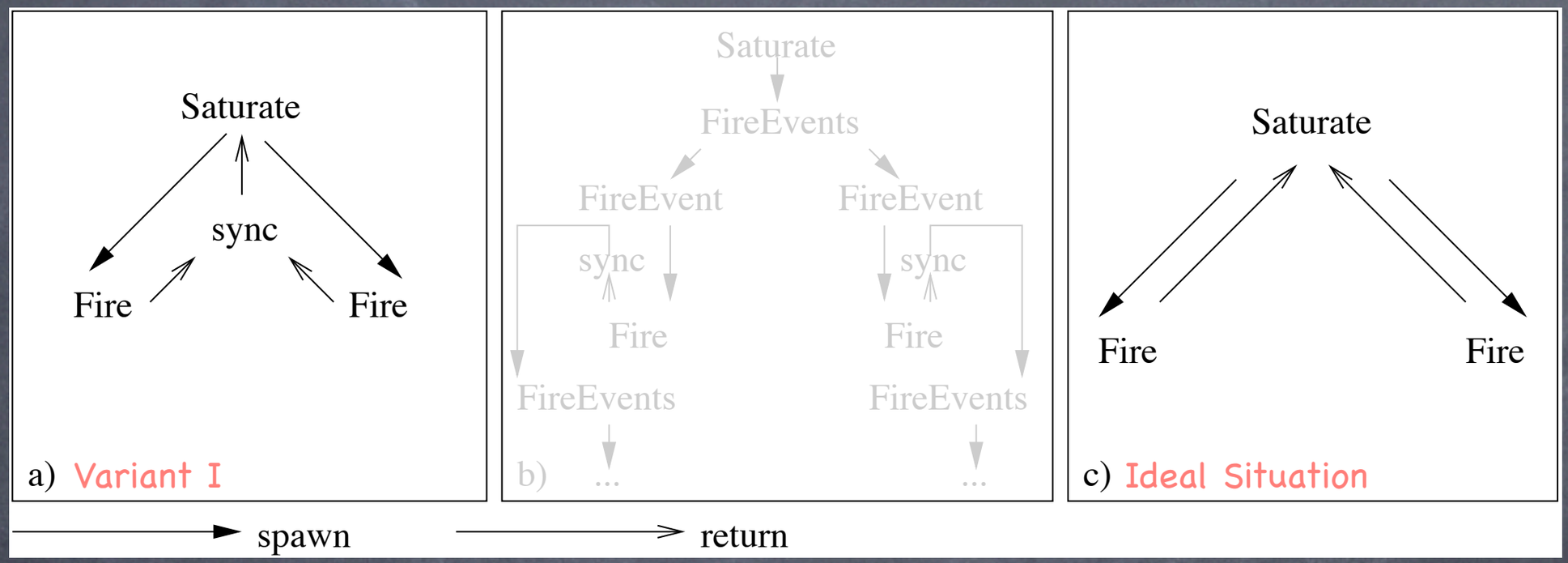
- Firing of events on a saturating node are parallelised
- Inlet handles computing unions and conducting in-place updates, as soon as a spawned firing returns
- Saturate function synchronises on the spawned firings; this is essential for
  - Determining when the firing loop shall terminate
  - Clearing up call stacks and thus controlling memory

Discussion


Saturate
sync
Fire
Fire
a) Variant I
Saturate
FireEvents
FireEvent
FireEvent
sync
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Fire
Fire
FireEvents
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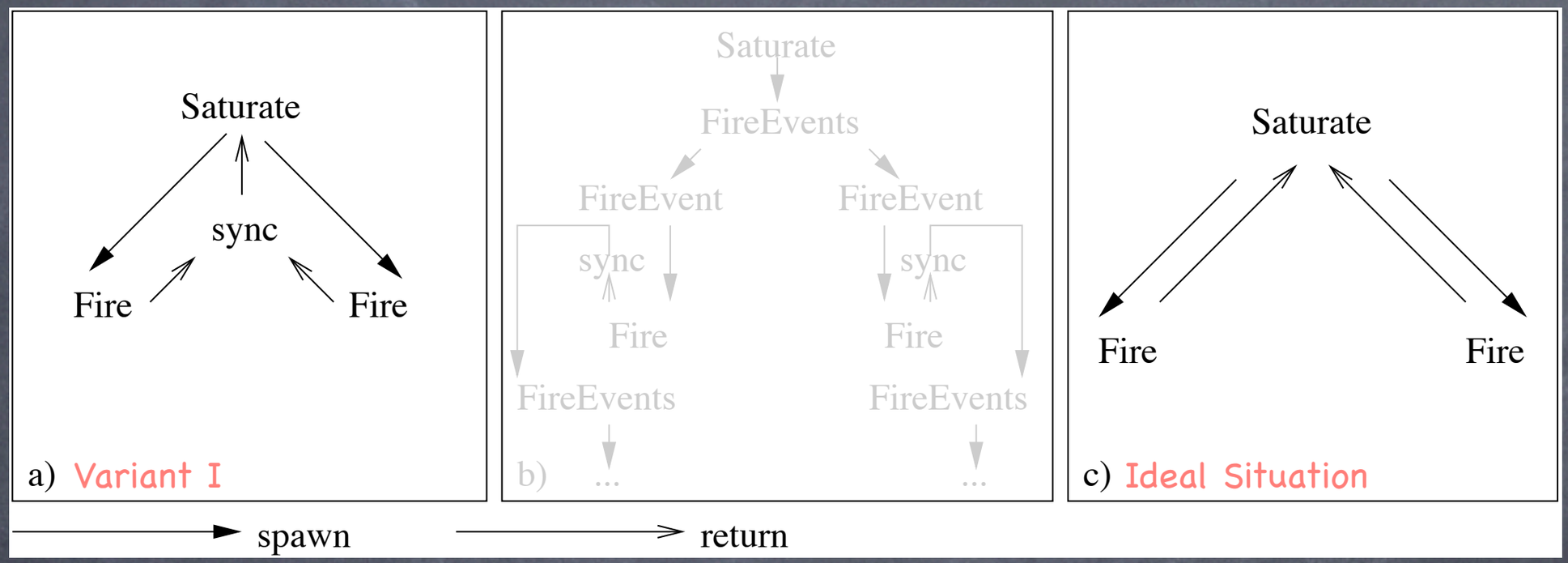
b) ...
...
Saturate
Fire
Fire
c) Ideal Situation
spawn
return

Discussion

- The different sizes of firing calls create load imbalance as parallel firings must synchronise before further firing



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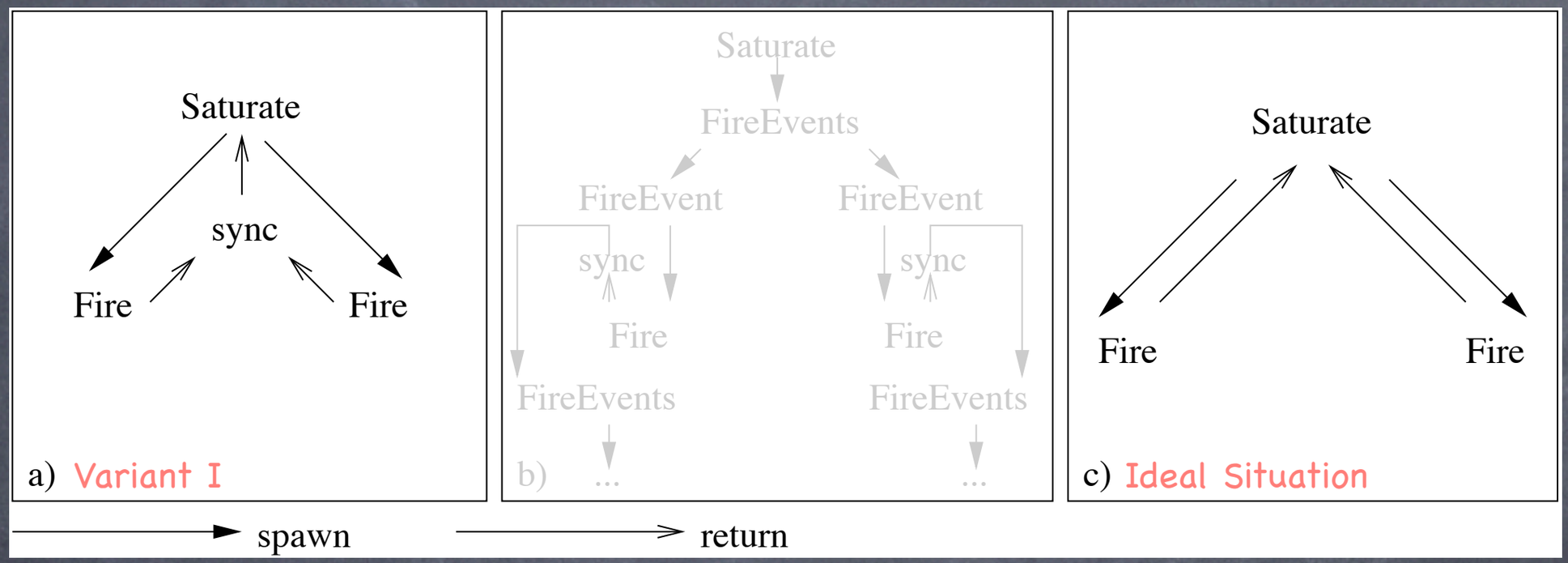

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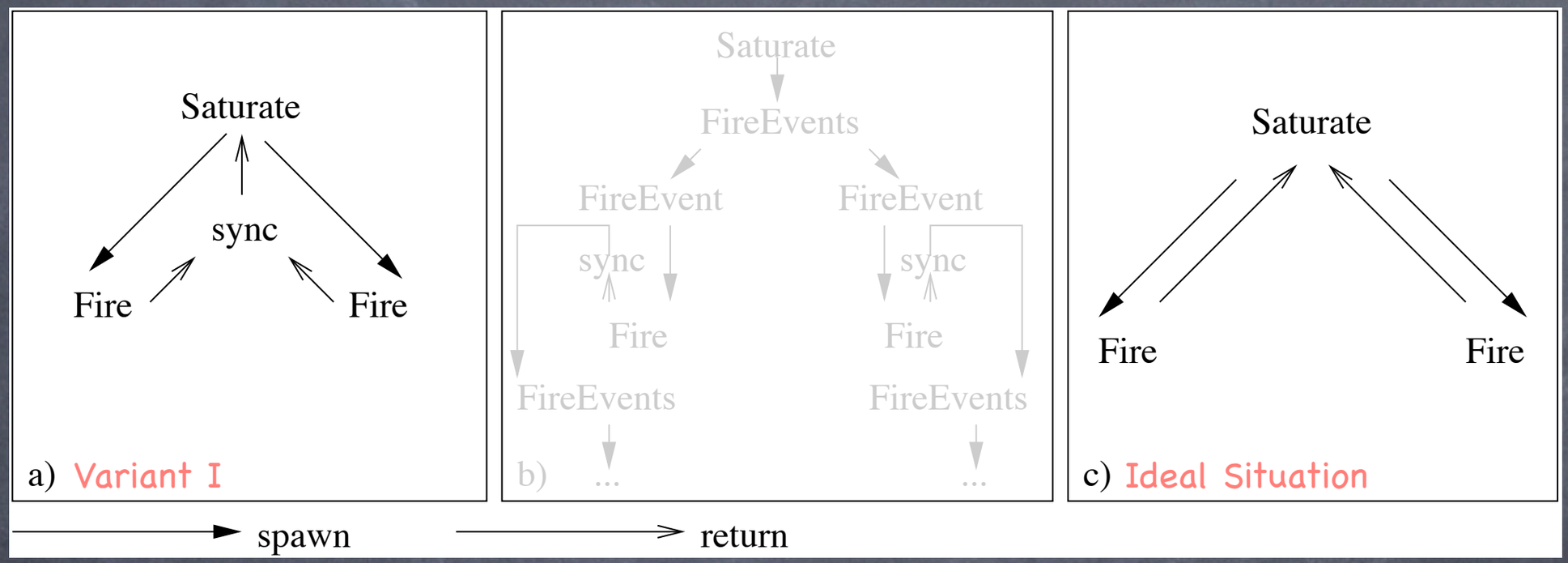

- It would be desirable to spawn more firings after a union, but inlets are not allowed to spawn functions

- Flags can also not be used to signal completed firings, as these would need to be checked within a busy-wait loop

Parallel Saturation in Cilk: Second Variant

cilk $Saturate$(in k:lvl, p:$node$)

Update p, a node at level k not in the hash table, in–place, to encode $\mathcal{N}^*_{\leq k}(\mathcal{B}(p))$.

declare $i : lcl$;

1. foreach $i \in \mathcal{S}_k$ do
2. if $p[i] \neq \mathbf{0}$ then
3. spawn $FireEvents(k, p, i)$;

cilk $FireEvents$(in k:lvl, p:$node$, i:lcl)

Fire e on $p[i]$ when $\mathcal{N}_{k,e}(i) \neq \mathbf{0}$.

declare $e : event$;

1. foreach $e \in \mathcal{E}_k$ do
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cilk $FireEvent$(in k:lvl, p:$node$, i:lcl, e:$event$)

Fire e on node $p[i]$ at level k.

declare $j : lcl$; $f : node$;

1. $f \Leftarrow Fire(e, k-1, p[i])$;
2. if $f \neq \mathbf{0}$ then
3. foreach $j \in \mathcal{N}_{k,e}(i)$ do
4. spawn $DoUnion(k, p, j, f)$;

cilk $DoUnion$(in k:lvl, p:$node$, j:lcl, f:$node$)

Fire events on $p[j]$ when $p[j]$ changes.

declare $u : node$;

1. $u \Leftarrow Union(k-1, f, p[j])$;
2. if $u \neq p[j]$ then
3. $p[j] \Leftarrow u$; spawn $FireEvents(k, p, j)$;

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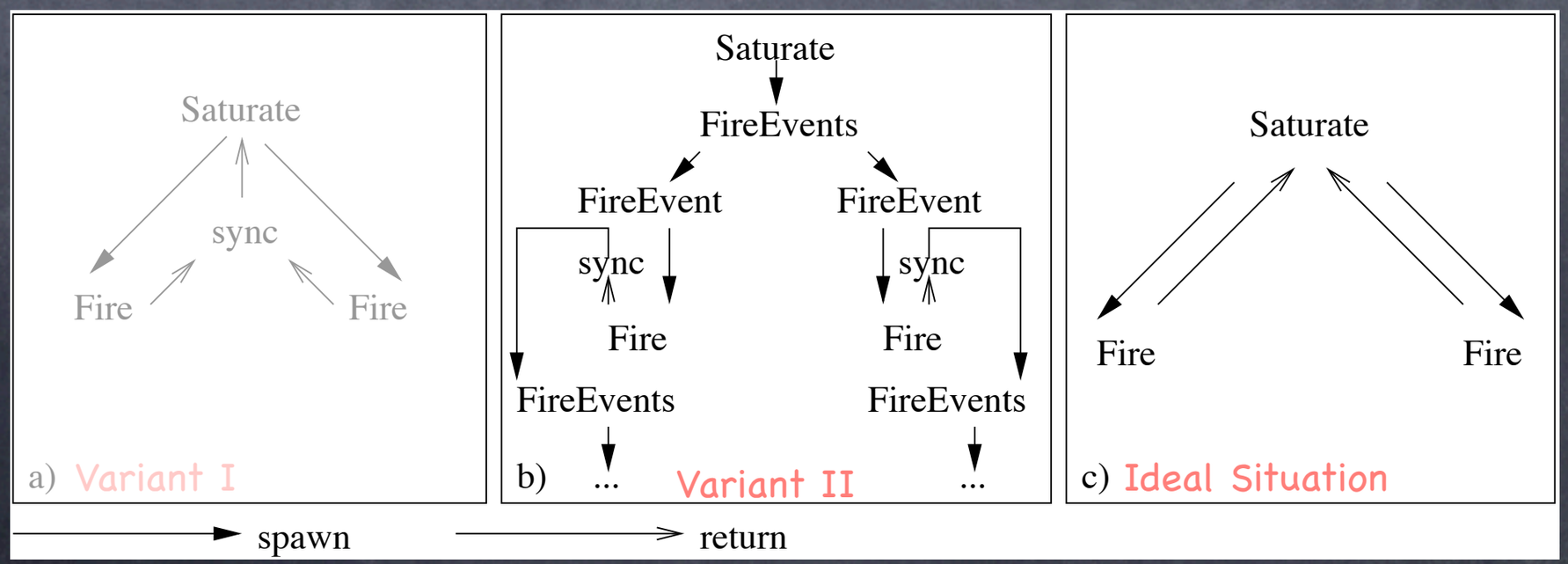

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➡ Hand-written version of Saturation using Pthreads ...

Parallel Saturation by Hand: Threadpool Version

Saturate(in k:*lvl*, p:*node*)

declare i : *lcl*;

1. foreach $i \in \mathcal{S}_k$ do
2. if $p[i] \neq \mathbf{0}$ then
3. $FireEvents(k, p, i)$;
4. if $Tasks(k, p) = 0$ then
5. $NodeSaturated(k, p)$;

FireEvents(in k:*lvl*, p:*node*, i:*lcl*)

declare e : *event*;

1. foreach $e \in \mathcal{E}_k$ do
2. if $\mathcal{N}_{k,e}(i) \neq \mathbf{0}$ then
3. $Fire(e, k, p, p[i], i)$;

Fire(in e:*event*, k:*lvl*, p:*node*, q:*node*, i:*lcl*):*node*

declare s : *node*; j : *lcl*;

...

4. $s{=}CreateNode(k{-}1)$;
5. foreach $j \in \mathcal{N}_{k,e}(i)$ do
6. $AddTask(k, p)$; $SetUpArc(k{-}1, s, j, p)$;

...

14. $AddQueue(Saturate(k{-}1, s))$;

...

NodeSaturated(in k:*lvl*, p:*node*)

declare q : *node*;

1. while $GetUpArc(k, p, i, q)$
2. $DoUnion(k{+}1, q, i, p)$;
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DoUnion spawns further FireEvents, if needed

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➡ "Pipelined" firings while managing memory and load balancing responsibly

Experimental Studies

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  - Industrial - NASA/Lockheed Martin's Runway Safety Monitor
- Implementation in C using Cilk/Pthreads running on
  - Dual-processor, dual-core PC (Xeon 3.06 GHz, 512 KB cache)
  - Redhat Linux AS4, kernel 2.6.9-22.ELsmp, glibc 2.3.4-2.13

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- The arguably most relevant model, NASA's Runway Safety Monitor, exhibits a superlinear speedup because of the additional positive effect of chaining

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  - Memory increase for Threadpool version - roughly 2-3x
  - Memory increase for Cilk version - roughly 10-20x
- The Cilk stack becomes huge!
  - This is the result of parallel Saturation relying on pipelining; but many verification algorithms are producer-consumer problems!
  - Our first variant of Saturation in Cilk, which adheres to the Cilk philosophy, is memory-efficient but hopelessly inefficient regarding run-time

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Maybe our time is better spent improving sequential algorithms (cf. BFS→Saturation) rather then parallelising them?!